

Graph of f

76. The graph of the function f shown above consists of two line segments and a semicircle. Let g be defined by

$g(x) = \int_0^x f(t) dt$. What is the value of $g(5)$?

- (A) 0 (B) $-1.5 + 2\pi$ (C) 2π (D) $1.5 + 2\pi$ (E) $4.5 + 2\pi$

$$\int_0^5 f(t) dt$$

77. The volume of a sphere is decreasing at a constant rate of 3 cubic centimeters per second. At the instant when the radius of the sphere is decreasing at a rate of 0.25 centimeter per second, what is the radius of the sphere?

(The volume V of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.)

(A) 0.141 cm

(B) 0.244 cm

(C) 0.250 cm

(D) 0.489 cm

(E) 0.977 cm

$$\frac{dV}{dt} = -3 \quad \frac{dr}{dt} = -\frac{1}{4} \quad r = ?$$

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$-3 = -\pi r^2$$

$$r^2 = \frac{3}{\pi}$$
$$= \sqrt{\frac{3}{\pi}}$$

78. Let f and g be continuous functions such that $\int_0^{10} f(x) \, dx = 21$, $\int_0^{10} \frac{1}{2}g(x) \, dx = 8$, and $\int_3^{10} (f(x) - g(x)) \, dx = 2$. What is the value of $\int_0^3 (f(x) - g(x)) \, dx$?

$$\frac{1}{2} \int = 8$$

(A) 3

(B) 7

(C) 11

(D) 15

(E) 19

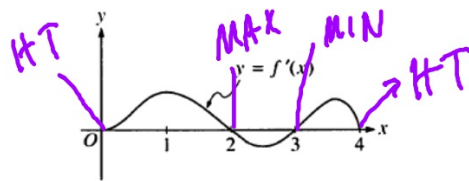
$$\int_0^{10} f = 21$$

$$\int_0^{10} g = 16$$

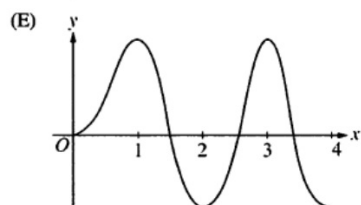
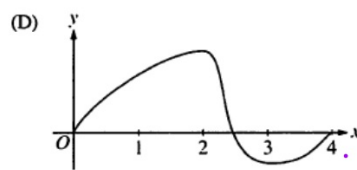
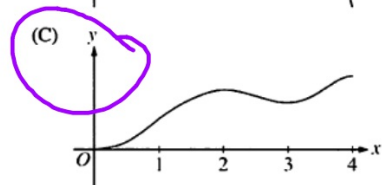
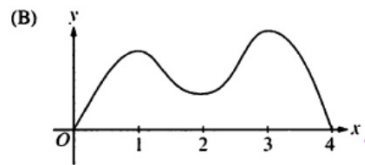
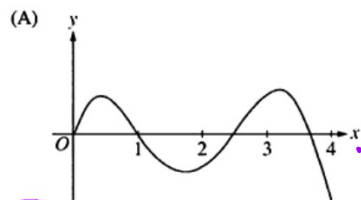
$$\int_0^{10} f - g = 5$$

$$\int_3^{10} f - g = 2$$

$$\therefore \int_0^3 = 3$$



79. The figure above shows the graph of f' , the derivative of the function f . If $f(0) = 0$, which of the following could be the graph of f ?



80. For time $t \geq 0$, the position of a particle traveling along a line is given by a differentiable function s . If s is increasing for $0 \leq t < 2$ and s is decreasing for $t > 2$, which of the following is the total distance the particle travels for $0 \leq t \leq 5$?

(A) $s(0) + \int_0^2 s'(t) dt - \int_2^5 s'(t) dt$

(B) $s(0) + \int_2^5 s'(t) dt - \int_0^2 s'(t) dt$

(C) $\int_2^5 s'(t) dt - \int_0^2 s'(t) dt$

(D) $\left| \int_0^5 s'(t) dt \right|$

(E) $\int_0^5 |s'(t)| dt$

$$\int_0^5 |v(t)| dt$$

$$\int_0^5 |s'(t)| dt$$

81. A cup of tea is cooling in a room that has a constant temperature of 70 degrees Fahrenheit ($^{\circ}\text{F}$). If the initial temperature of the tea, at time $t = 0$ minutes, is 200°F and the temperature of the tea changes at the rate $R(t) = -6.89e^{-0.053t}$ degrees Fahrenheit per minute, what is the temperature, to the nearest degree, of the tea after 4 minutes?

- (A) 175°F (B) 130°F (C) 95°F (D) 70°F (E) 45°F

$$200 + \int_0^4 R(t) dt = 175.165$$

82. The derivative of the function f is given by $f'(x) = x^3 - 4\sin(x^2) + 1$. On the interval $(-2.5, 2.5)$, at which of the following values of x does f have a relative maximum?

- (A) -1.970 and 0
- (B) -1.467 and 1.075
- (C) -0.475 , 0.542 , and 1.396
- (D) -0.475 and 1.396 only
- (E) 0.542 only

graph f' $-2.5 \rightarrow 2.5$
ABOVE TO BELOW
.542

x	0	0.5	1	1.5	2	2.5	3
$f(x)$	0	4	10	18	28	40	54

83. The table above gives selected values for a continuous function f . If f is increasing over the closed interval $[0, 3]$, which of the following could be the value of $\int_0^3 f(x) dx$?

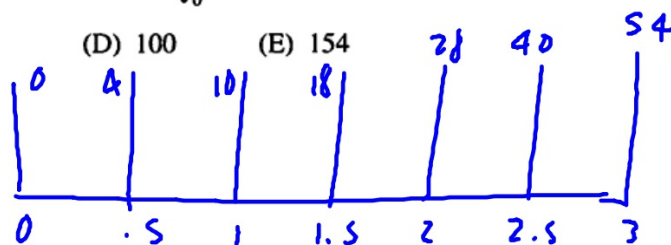
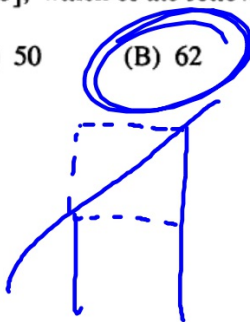
(A) 50

(B) 62

(C) 77

(D) 100

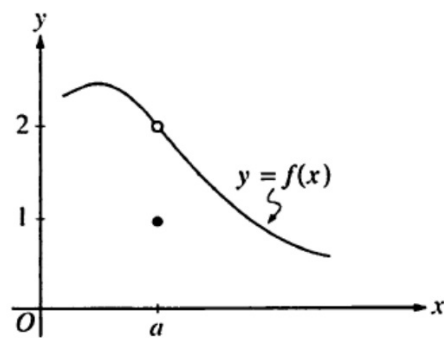
(E) 154



$$L = 0 + 2 + 5 + 9 + 14 + 20 = 50$$

$$R = 2 + 5 + 9 + 14 + 20 + 27 = 77$$

$$50 < ? < 77$$



84. The graph of a function f is shown in the figure above. Which of the following statements is true?

- (A) $f(a) = 2$ ~~X~~
- (B) f is continuous at $x = a$. ~~X~~
- (C) $\lim_{x \rightarrow a} f(x) = 1$ ~~X~~
- (D) $\lim_{x \rightarrow a} f(x) = 2$
- (E) $\lim_{x \rightarrow a} f(x)$ does not exist.

85. A particle moves along the x -axis so that at time $t \geq 0$ its position is given by $x(t) = \cos \sqrt{t}$. What is the velocity of the particle at the first instance the particle is at the origin?

- (A) -1 (B) -0.624 (C) -0.318 (D) 0 (E) 0.065

$$x(t) = 0$$

$$y_1 = x(t)$$

$$y_2 = d(y_1(x), x)$$

$$y_2(x_0)$$

86. If $f'(x) > 0$ for all x and $f''(x) < 0$ for all x , which of the following could be a table of values for f ?

(A)

x	$f(x)$
-1	4
0	3
1	1

(B)

x	$f(x)$
-1	4
0	4
1	4

(C)

x	$f(x)$
-1	4
0	5
1	6

(D)

x	$f(x)$
-1	4
0	5
1	7

(E)

x	$f(x)$
-1	4
0	6
1	7

f is inc CD

values up but
by less and less

87. Let f be the function with first derivative given by $f'(x) = (3 - 2x - x^2)\sin(2x - 3)$. How many relative extrema does f have on the open interval $-4 < x < 2$?

(A) Two

(B) Three

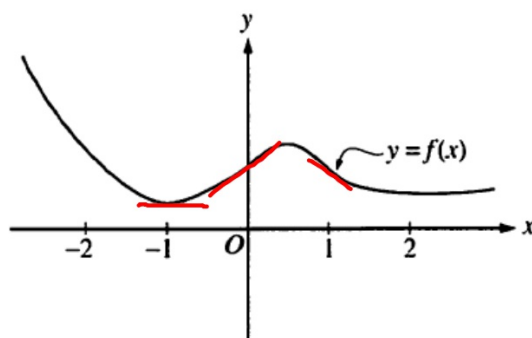
(C) Four

(D) Five

(E) Six

Careful

Zoom in on close
issues



88. The graph of a twice-differentiable function f is shown in the figure above. Which of the following is true?

(A) $f'(-1) < f'(1) < f'(0)$

(B) $f'(-1) < f'(0) < f'(1)$

(C) $f'(0) < f'(-1) < f'(1)$

(D) $f'(1) < f'(-1) < f'(0)$

(E) $f'(1) < f'(0) < f'(-1)$

$$f'(-1) = 0$$

$$f'(0) > 0$$

$$f'(1) < 0$$

$$f'(1) < f'(-1) < f'(0)$$

89. What is the volume of the solid generated when the region bounded by the graph of $x = \sqrt{y-2}$ and the lines $x = 0$ and $y = 5$ is revolved about the y -axis?

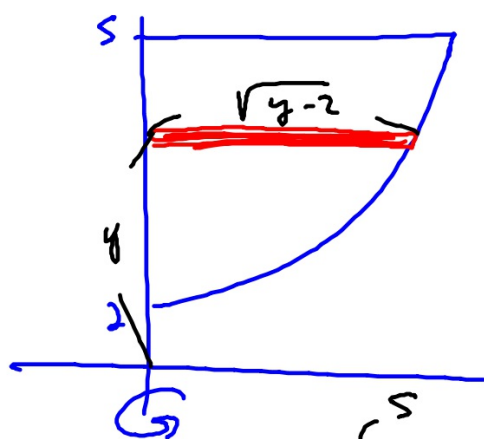
(A) 3.464

(B) 4.500

(C) 7.854

(D) 10.883

(E) 14.137



$$V = \pi \int_2^5 (\sqrt{y-2})^2 dy$$

$x^2 = y - 2$
 $y = x^2 + 2$

90. The population P of a city grows according to the differential equation $\frac{dP}{dt} = kP$, where k is a constant and t is measured in years. If the population of the city doubles every 12 years, what is the value of k ?
- (A) 0.058 (B) 0.061 (C) 0.167 (D) 0.693 (E) 8.318

$$k = \frac{\ln 2}{12} = .058$$

$$\begin{aligned} y &= y_0 e^{kt} \\ 2 &= 1 e^{12k} \\ \ln 2 &= 12k \\ \frac{\ln 2}{12} &= k \end{aligned}$$

Exp. growth

$$\begin{aligned} \frac{dy}{dt} &= ky \\ y &= y_0 e^{kt} \\ t &= \frac{\ln 2}{k} \\ k &= \frac{\ln 2}{t} \end{aligned}$$

91. The function f is continuous and $\int_0^8 f(u) \, du = 6$. What is the value of $\int_1^3 \underbrace{xf(x^2 - 1)} \, dx$?

(A) $\frac{3}{2}$

(B) 3

(C) 6

(D) 12

(E) 24

$$u = x^2 - 1$$

$$du = 2x \, dx$$

$$\frac{1}{2} du = \underbrace{x \, dx}$$

$$x=1 \quad u=0$$

$$x=3 \quad u=8$$

$$\frac{1}{2} \int_0^8 f(u) \, du$$

$$\frac{1}{2} (6)$$

$$3$$

92. The function f is defined for all x in the closed interval $[a, b]$. If f does not attain a maximum value on $[a, b]$, which of the following must be true?

(A) f is not continuous on $[a, b]$.

(B) f is not bounded on $[a, b]$.

(C) f does not attain a minimum value on $[a, b]$.

(D) The graph of f has a vertical asymptote in the interval $[a, b]$.

(E) The equation $f'(x) = 0$ does not have a solution in the interval $[a, b]$.