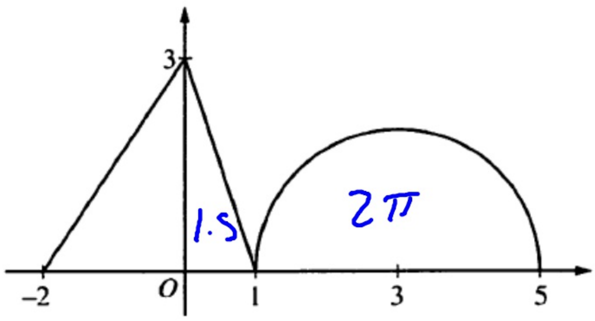


Graph of f

76. The graph of the function f shown above consists of two line segments and a semicircle. Let g be defined by $g(x) = \int_0^x f(t)\,dt$. What is the value of $g(5)$?

(A) 0 (B) $-1.5 + 2\pi$ (C) 2π (D) $1.5 + 2\pi$ (E) $4.5 + 2\pi$

$$\int_0^5 f(t)\,dt =$$

77. The volume of a sphere is decreasing at a constant rate of 3 cubic centimeters per second. At the instant when the radius of the sphere is decreasing at a rate of 0.25 centimeter per second, what is the radius of the sphere?

(The volume V of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.)

(A) 0.141 cm (B) 0.244 cm (C) 0.250 cm (D) 0.489 cm (E) 0.977 cm

$\frac{dV}{dt} = -3$ $\frac{dr}{dt} = -\frac{1}{4}$

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$-3 = -\pi r^2$

$r = \sqrt{\frac{3}{\pi}}$

78. Let f and g be continuous functions such that $\int_0^{10} f(x)\,dx = 21$, $\int_0^{10} \frac{1}{2}g(x)\,dx = 8$, and $\int_3^{10} (f(x) - g(x))\,dx = 2$. What is the value of $\int_0^3 (f(x) - g(x))\,dx$?

(A) 3 (B) 7 (C) 11 (D) 15 (E) 19

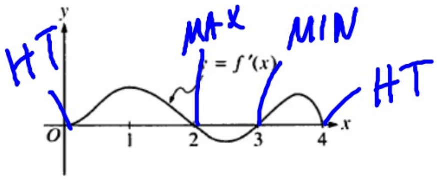
$$\int_0^{10} f = 21$$

$$\int_0^{10} g = 16$$

$$\int_0^{10} f - g = 5$$

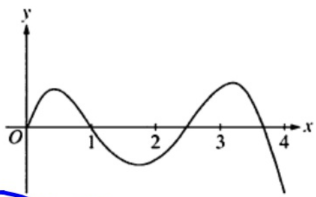
$$\int_3^{10} f - g = 2$$

$$\int_0^{3} = 3$$

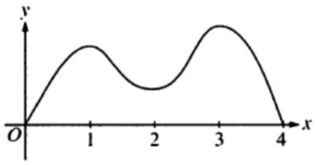


79. The figure above shows the graph of f', the derivative of the function f. If $f(0) = 0$, which of the following could be the graph of f?

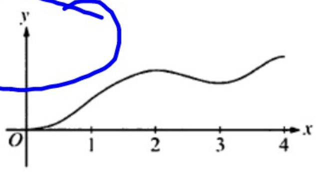
(A)



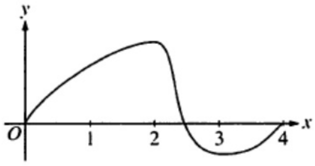
(B)



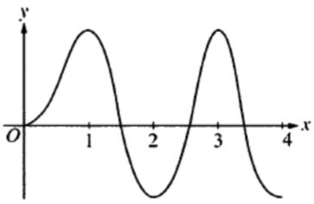
(C)



(D)



(E)



80. For time $t \geq 0$, the position of a particle traveling along a line is given by a differentiable function s. If s is increasing for $0 \leq t < 2$ and s is decreasing for $t > 2$, which of the following is the total distance the particle travels for $0 \leq t \leq 5$?

(A) $s(0) + \int_0^2 s'(t)\,dt - \int_2^5 s'(t)\,dt$

(B) $s(0) + \int_2^5 s'(t)\,dt - \int_0^2 s'(t)\,dt$

(C) $\int_2^5 s'(t)\,dt - \int_0^2 s'(t)\,dt$

(D) $\left|\int_0^5 s'(t)\,dt\right|$

(E) $\int_0^5 |s'(t)|\,dt$

$\int_0^5 |v(t)|\,dt$

$\int_0^5 |s'(t)|\,dt$

81. A cup of tea is cooling in a room that has a constant temperature of 70 degrees Fahrenheit (°F). If the initial temperature of the tea, at time $t = 0$ minutes, is 200°F and the temperature of the tea changes at the rate $R(t) = -6.89e^{-0.053t}$ degrees Fahrenheit per minute, what is the temperature, to the nearest degree, of the tea after 4 minutes?

(A) 175°F (B) 130°F (C) 95°F (D) 70°F (E) 45°F

$$200 + \int_0^4 R(t)\,dt = 175.165$$

82. The derivative of the function f is given by $f'(x) = x^3 - 4\sin\left(x^2\right) + 1$. On the interval $(-2.5, 2.5)$, at which of the following values of x does f have a relative maximum?

(A) -1.970 and 0

(B) -1.467 and 1.075

(C) -0.475, 0.542, and 1.396

(D) -0.475 and 1.396 only

(E) 0.542 only

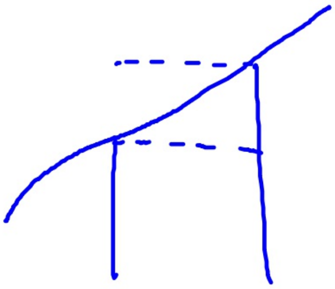
graph f' on -2.5 to 2.5

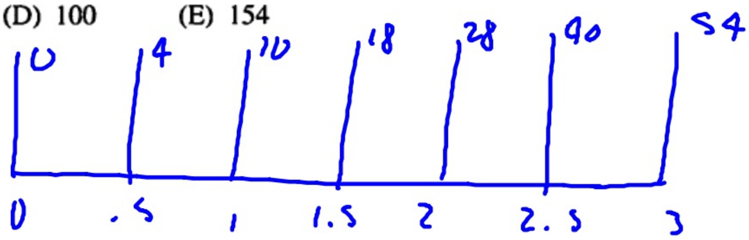
look for above to below

x	0	0.5	1	1.5	2	2.5	3
$f(x)$	0	4	10	18	28	40	54

83. The table above gives selected values for a continuous function f. If f is increasing over the closed interval $[0, 3]$, which of the following could be the value of $\int_0^3 f(x)\,dx$?

(A) 50 (B) 62 (C) 77 (D) 100 (E) 154

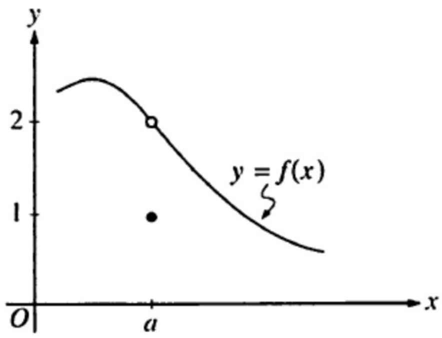




$0 + 2 + 5 + 9 + 14 + 20 = 50$

$2 + 5 + 9 + 14 + 20 + 27 = 77$

$50 < ? < 77$



84. The graph of a function f is shown in the figure above. Which of the following statements is true?

(A) $f(a) = 2$

(B) f is continuous at $x = a$.

(C) $\lim_{x \to a} f(x) = 1$

(D) $\lim_{x \to a} f(x) = 2$

(E) $\lim_{x \to a} f(x)$ does not exist.

85. A particle moves along the x-axis so that at time $t \geq 0$ its position is given by $x(t) = \cos\sqrt{t}$. What is the velocity of the particle at the first instance the particle is at the origin?

(A) -1 (B) -0.624 (C) -0.318 (D) 0 (E) 0.065

$x(t) = 0$ sto xc

y2(xc)

86. If $f'(x) > 0$ for all x and $f''(x) < 0$ for all x, which of the following could be a table of values for f?

(A)

x	$f(x)$
–1	4
0	3
1	1

(B)

x	$f(x)$
–1	4
0	4
1	4

(C)

x	$f(x)$
–1	4
0	5
1	6

(D)

x	$f(x)$
–1	4
0	5
1	7

(E)

x	$f(x)$
–1	4
0	6
1	7

$f \uparrow$ CD

Values go up –
but by less
and less

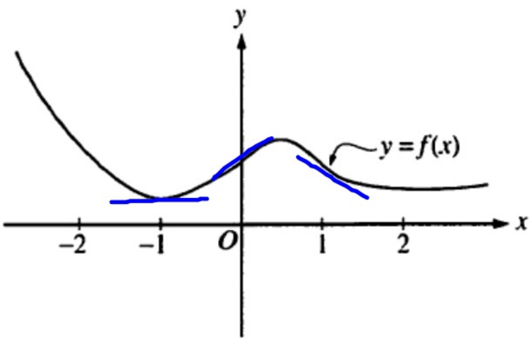
87. Let f be the function with first derivative given by $f'(x) = \left(3 - 2x - x^2\right)\sin(2x - 3)$. How many relative extrema does f have on the open interval $-4 < x < 2$?

(A) Two (B) Three (C) Four (D) Five (E) Six

graph on $-4 \rightarrow 2$

CAREFUL

Zoom in close spots.



88. The graph of a twice-differentiable function f is shown in the figure above. Which of the following is true?

(A) $f'(-1) < f'(1) < f'(0)$

(B) $f'(-1) < f'(0) < f'(1)$

(C) $f'(0) < f'(-1) < f'(1)$

(D) $f'(1) < f'(-1) < f'(0)$

(E) $f'(1) < f'(0) < f'(-1)$

$f'(-1) = 0$

$f'(0) > 0$

$f'(1) < 0$

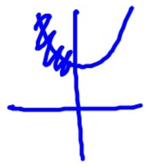
$f'(1) < f'(-1) < f'(0)$

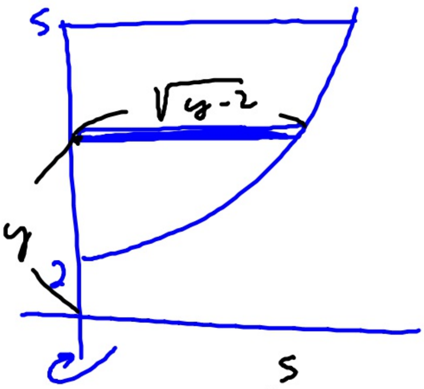
89. What is the volume of the solid generated when the region bounded by the graph of $x = \sqrt{y-2}$ and the lines $x = 0$ and $y = 5$ is revolved about the y-axis?

(A) 3.464 (B) 4.500 (C) 7.854 (D) 10.883 (E) 14.137

$x^2 = y - 2$

$y = x^2 + 2$





$$V = \pi \int_2^5 \left(\sqrt{y-2}\right)^2 \, dy = 14.137$$

90. The population P of a city grows according to the differential equation $\frac{dP}{dt} = kP$, where k is a constant and t is measured in years. If the population of the city doubles every 12 years, what is the value of k?

(A) 0.058 (B) 0.061 (C) 0.167 (D) 0.693 (E) 8.318

Exp Growth

$k = \frac{\ln 2}{12} =$

$y = y_0 e^{kt}$

$t = \frac{\ln 2}{k}$ $k = \frac{\ln 2}{t}$

$\frac{dy}{dt} = ky$

91. The function f is continuous and $\int_0^8 f(u)\,du = 6$. What is the value of $\int_1^3 xf\left(x^2 - 1\right) dx$?

(A) $\frac{3}{2}$ (B) 3 (C) 6 (D) 12 (E) 24

$u = x^2 - 1$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$x = 1 \quad u = 0$

$x = 3 \quad u = 8$

$\frac{1}{2}\int_0^8 f(u)\,du$

$\frac{1}{2}\left(6\right) = 3$

92. The function f is defined for all x in the closed interval $[a, b]$. If f does not attain a maximum value on $[a, b]$, which of the following must be true?

(A) f is not continuous on $[a, b]$.

(B) f is not bounded on $[a, b]$.

(C) f does not attain a minimum value on $[a, b]$.

(D) The graph of f has a vertical asymptote in the interval $[a, b]$.

(E) The equation $f'(x) = 0$ does not have a solution in the interval $[a, b]$.