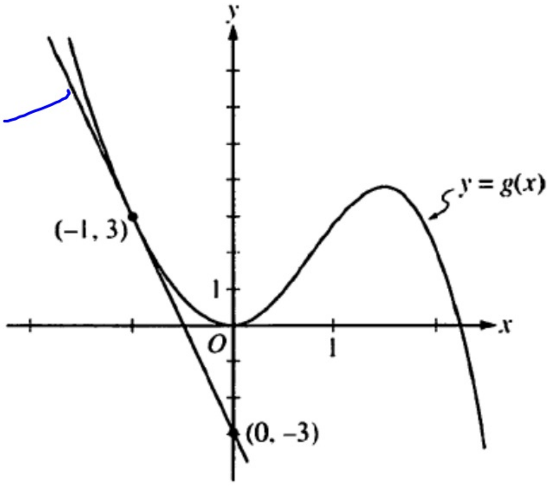
$M = -6$



19. The figure above shows the graph of the function g and the line tangent to the graph of g at $x = -1$. Let h be the function given by $h(x) = e^x \cdot g(x)$. What is the value of $h'(-1)$?

(A) $\frac{9}{e}$ (B) $\frac{-3}{e}$ (C) $\frac{-6}{e}$ (D) $\frac{-6}{e} - \frac{3}{e^2}$ (E) -6

$h'(x) = e^x g'(x) + e^x g(x)$ $= \frac{1}{e}\left[-6 + 3\right] = -\frac{3}{e}$

$= e^x(g'(x) + g(x))$

$h'(-1) = e^{-1}(g'(-1) + g(-1))$

20. For $x > 0$, $\frac{d}{dx}\left(\int_0^{2x} \ln\left(t^3+1\right)\,dt\right) =$

(A) $\ln\left(x^3+1\right)$

(B) $\ln\left(8x^3+1\right)$

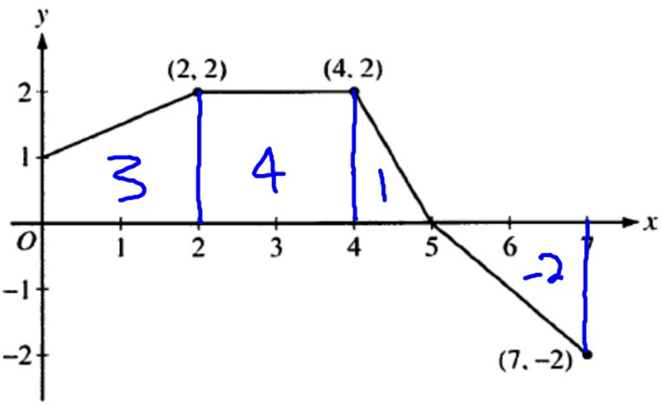
(C) $2\ln\left(x^3+1\right)$

(D) $2\ln\left(8x^3+1\right)$

(E) $24x^2\ln\left(8x^3+1\right)$

$\underline{2\ln(8x^3+1)} - (\quad)(0)$

$\frac{1}{2}(3)2$



Graph of f

21. The graph of a function f is shown above. What is the value of $\int_0^7 f(x)\,dx$?

(A) 6 (B) 8 (C) 10 (D) 14 (E) 18

22. The function f is continuous for all real numbers, and the average rate of change of f on the closed interval $[6, 9]$ is $-\frac{3}{2}$. For $6 < c < 9$, there is no value of c such that $f'(c) = -\frac{3}{2}$. Of the following, which must be true?

(A) $\frac{1}{3}\int_6^9 f(x)\,dx = -\frac{3}{2}$

(B) $\int_6^9 f(x)\,dx$ does not exist.

(C) $\frac{f'(6) + f'(9)}{2} = -\frac{3}{2}$

(D) $f'(x) < 0$ for all x in the open interval $(6, 9)$.

(E) f is not differentiable on the open interval $(6, 9)$.

23. Let f be the function defined by $f(x) = 2x + e^x$. If $g(x) = f^{-1}(x)$ for all x and the point $(0, 1)$ is on the graph of f, what is the value of $g'(1)$?

(A) $\frac{1}{2+e}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) 3 (E) $2 + e$

$(f^{-1})'(1)$

Since $(0,1)$ on f $\rightarrow$ $(f^{-1})'(1) = \frac{1}{f'(0)}$

$f'(x) = 2 + e^x$

$f'(0) = 2 + e^0 = 3$

$\frac{1}{3}$

24. The function g is given by $g(x) = 4x^3 + 3x^2 - 6x + 1$. What is the absolute minimum value of g on the closed interval $[-2, 1]$?

(A) -7 (B) $-\frac{3}{4}$ (C) 0 (D) 2 (E) 6

$g'(x) = 12x^2 + 6x - 6$

$2x^2 + x - 1 = 0$

$\frac{5}{4} - 2$

$(2x - 1)(x + 1) = 0$

$\frac{5}{4} - \frac{8}{4}$

$x = \frac{1}{2}$ or $x = -1$

$g(-2) = -32 + 12 + 12 + 1 = -7$

$g(1) = 4 + 3 - 6 + 1 = 2$

$g(\frac{1}{2}) = \frac{1}{2} + \frac{3}{4} - 3 + 1 = -\frac{3}{4}$

$g(-1) = -4 + 3 + 6 + 1 = 6$

25. Which of the following is the solution to the differential equation $\frac{dy}{dx} = e^{y+x}$ with the initial condition $y(0) = -\ln 4$?

(A) $y = -x - \ln 4$

(B) $y = x - \ln 4$

(C) $y = -\ln\left(-e^x + 5\right)$

(D) $y = -\ln\left(e^x + 3\right)$

(E) $y = \ln\left(e^x + 3\right)$

$$\frac{dy}{dx} = e^y e^x$$

$$e^{-y}\,dy = e^x\,dx$$

$$-e^{-y} = e^x + C$$

$$-\frac{1}{e^y} = e^x + C$$

$$-4 = 1 + C$$

$$-5 = C$$

$$-\frac{1}{e^y} = e^x - 5$$

$$\frac{1}{e^y} = 5 - e^x$$

$$e^y = \frac{1}{5 - e^x}$$

$$y = \ln\left(\frac{1}{5 - e^x}\right)$$

$$= \ln\left(5 - e^x\right)^{-1}$$

$$= -\ln\left(5 - e^x\right)$$

$$-\frac{1}{\frac{1}{4}}$$

$$-\frac{1}{e^{-\ln 4}}$$

$$-\frac{1}{e^{\ln 1/4}}$$

26. Which of the following is an antiderivative of $f(x) = \sqrt{1 + x^3}$?

(A) $\frac{2}{3}\left(1 + x^3\right)^{3/2}$

(B) $\dfrac{\frac{2}{3}\left(1 + x^3\right)^{3/2}}{3x^2}$

(C) $\int_0^{1+x^3} \sqrt{t}\, dt$

(D) $\int_0^{x^3} \sqrt{1 + t}\, dt$

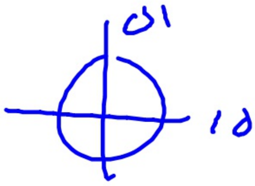
(E) $\int_0^{x} \sqrt{1 + t^3}\, dt$

$\frac{d}{dx}\int_0^x \sqrt{1+t^3}\,dt = \sqrt{1+x^3}$ ✓

27. For time $t \geq 0$, the height h of an object suspended from a spring is given by $h(t) = 16 + 7\cos\left(\frac{\pi t}{4}\right)$. What is the average height of the object from $t = 0$ to $t = 2$?

(A) 16 (B) $\frac{39}{2}$ (C) $16 - \frac{14}{\pi}$ (D) $16 + \frac{14}{\pi}$ (E) $32 + \frac{28}{\pi}$

$$\frac{1}{2}\int_0^2 \left[16 + 7\cos\frac{\pi}{4}t\right] dt$$



$$\frac{1}{2}\left[16t + 7\frac{4}{\pi}\sin\frac{\pi}{4}t\right]_0^2$$

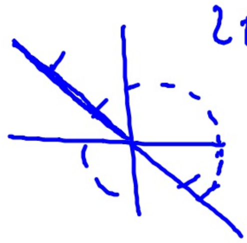
$$\frac{1}{2}\left[\left(32 + \frac{28}{\pi}\sin\frac{\pi}{2}\right) - \cancel{\left(\frac{28}{\pi}\sin 0\right)}\right]$$

$$16 + \frac{14}{\pi}$$

28. The function f is defined by $f(x) = \sin x + \cos x$ for $0 \le x \le 2\pi$. What is the x-coordinate of the point of inflection where the graph of f changes from concave down to concave up?

(A) $\frac{\pi}{4}$ (B) $\frac{3\pi}{4}$ (C) $\frac{5\pi}{4}$ (D) $\frac{7\pi}{4}$ (E) $\frac{9\pi}{4}$

$2\pi - \frac{\pi}{4}$



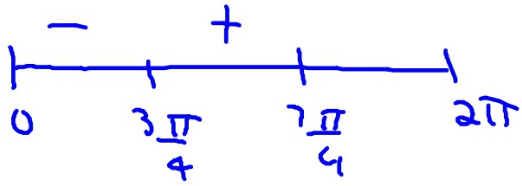
$f'(x) = \cos x - \sin x$

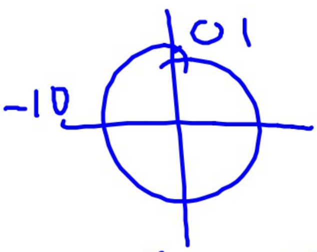
$f''(x) = -\sin x - \cos x$

$f''(x) = 0 \rightarrow -\sin x = \cos x$

$x = \frac{3\pi}{4}$ or $x = \frac{7\pi}{4}$

$-\sin\frac{\pi}{2} - \cos\frac{\pi}{2}$





$-\cancel{\sin}\,\pi - \cos\pi$