(b) For $y \leq 11$, find the y-coordinate of each point on the graph where the line tangent to the graph at that point is vertical.

$1 - 5\sin y = 0 \rightarrow y = .201$ or $y = 6.485$ or $y = 9.223$

(c) Find the average value of the x-coordinates of the points on the graph in the first quadrant between $y = 5$ and $y = 9$.

$$x^2 = -2 + y + 5\cos y$$

$$x = \sqrt{-2 + y + 5\cos y}$$

$$\frac{1}{9-5}\int_5^9 \sqrt{-2 + y + 5\cos y}\, dy = 2.550$$

t (seconds)	0	3	5	8	12
$k(t)$ (feet per second)	0	5	10	20	24

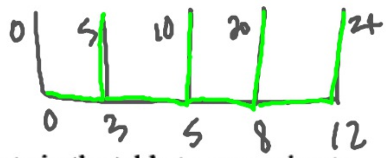
3. Kathleen skates on a straight track. She starts from rest at the starting line at time $t = 0$. For $0 < t \le 12$ seconds, Kathleen's velocity k, measured in feet per second, is differentiable and increasing. Values of $k(t)$ at various times t are given in the table above.

(a) Use the data in the table to estimate Kathleen's acceleration at time $t = 4$ seconds. Show the computations that lead to your answer. Indicate units of measure.

$$a(4) = k'(4) = \frac{k(5) - k(3)}{5-3} = \frac{10-5}{2} = \frac{5}{2}$$

$\therefore \frac{5}{2}$ ft/sec/sec

t (seconds)	0	3	5	8	12
$k(t)$ (feet per second)	0	5	10	20	24



(b) Use a right Riemann sum with the four subintervals indicated by the data in the table to approximate $\int_0^{12} k(t)\,dt$. Indicate units of measure. Is this approximation an overestimate or an underestimate for the value of $\int_0^{12} k(t)\,dt$? Explain your reasoning.

$$\int_0^{12} h(t)\,dt \approx 5(3) + 10(2) + 20(3) + 24(4) = 191$$

$\therefore$ 191 feet

This approximation is an overestimate because k is increasing on $(0,12)$.

(c) Nathan skates on the same track, starting 5 feet ahead of Kathleen at time $t = 0$. Nathan's velocity, in feet per second, is given by $n(t) = \dfrac{150}{t+3} - 50e^{-t}$. Write, but do not evaluate, an expression involving an integral that gives Nathan's distance from the starting line at time $t = 12$ seconds.

$$5 + \int_0^{12} n(t)\,dt$$

(d) Write an expression for Nathan's acceleration in terms of t.

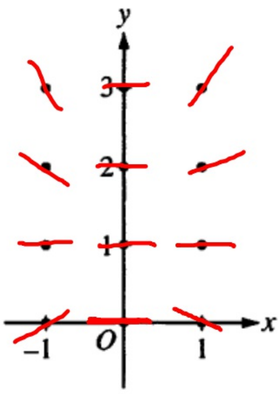
$$n(t) = 150(t+3)^{-1} - 50e^{-t}$$

$$n'(t) = -150(t+3)^{-2}(1) - 50e^{-t}(-1)$$

$$= -\frac{150}{(t+3)^2} + 50e^{-t}$$

4. Consider the differential equation $\frac{dy}{dx} = \frac{x(y-1)}{4}$.

(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.



(b) Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(1) = 3$. Write an equation for the line tangent to the graph of f at the point $(1, 3)$ and use it to approximate $f(1.4)$.

$$\left.\frac{dy}{dx}\right|_{(1,3)} = \frac{1}{2}$$

$$y - 3 = \frac{1}{2}(x-1)$$

$$y = 3 + \frac{1}{2}(x-1)$$

$$f(1.4) \approx y(1.4) = 3 + \frac{1}{2}(1.4-1) = 3.2$$

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(1) = 3$.

$$\frac{dy}{dx} = \frac{x(y-1)}{4}$$

$$\frac{1}{y-1}\,dy = \frac{1}{4}x\,dx$$

$$\ln|y-1| = \frac{1}{8}x^2 + C$$

$$y-1 = e^{\frac{1}{8}x^2+C}$$

$$y-1 = Ae^{\frac{1}{8}x^2}$$

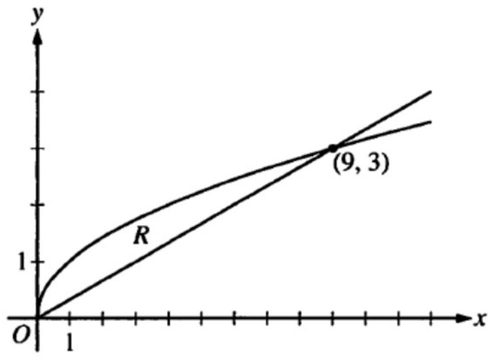
$$y = 1 + Ae^{\frac{1}{8}x^2}$$

$$3 = 1 + Ae^{1/8}$$

$$\frac{2}{e^{1/8}} = A = 2e^{-1/8}$$

$$y = 1 + 2e^{-\frac{1}{8}}e^{\frac{1}{8}x^2} \quad ✓$$

$$y = 1 + 2e^{\frac{x^2-1}{8}}$$



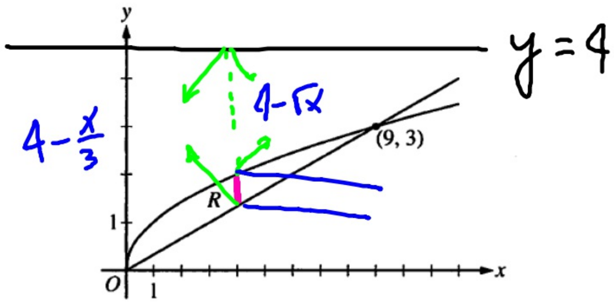
5. Let R be the region in the first quadrant enclosed by the graphs of $g(x) = \sqrt{x}$ and $h(x) = \frac{x}{3}$, as shown in the figure above.

(a) Find the area of region R.

$$A = \int_0^9 \left(\sqrt{x} - \frac{1}{3}x\right) dx = \left[\frac{2}{3}x^{3/2} - \frac{1}{6}x^2\right]_0^9$$

$$= \left(18 - \frac{81}{6}\right) - \left(0\right)$$

$$= \frac{9}{2}$$



(b) Write, but do not evaluate, an expression involving one or more integrals that gives the volume of the solid generated when R is revolved about the horizontal line $y = 4$.

$$V = \pi \int_0^9 \left[\left(4 - \frac{x}{3}\right)^2 - \left(4 - \sqrt{x}\right)^2 \right] dx$$

(c) Find the maximum vertical distance between the graph of g and the graph of h between $x = 0$ and $x = 16$. Justify your answer.

$$D(x) = \sqrt{x} - \frac{1}{3}x$$

$$D'(x) = \frac{1}{2\sqrt{x}} - \frac{1}{3} = \frac{3-2\sqrt{x}}{6\sqrt{x}}$$

$$D'(x) = 0 \rightarrow 3 - 2\sqrt{x} = 0$$
$$2\sqrt{x} = 3$$
$$x = \frac{9}{4}$$

$$D(0) = 0$$
$$D(16) = -\frac{4}{3} \quad \left(\frac{4}{3}\right)$$
$$D\left(\frac{9}{4}\right) = \frac{3}{4}$$

Since D is contnm on [0,16] by EVT max val ~ $\frac{4}{3}$.

6. Let $g(x) = 4(x+1)^{-2/3}$ and let f be the function defined by $f(x) = \int_0^x g(t)\,dt$ for $x \geq 0$.

(a) Find $f(26)$.

$$f(26) = \int_0^{26} 4(x+1)^{-2/3}\,dx$$

$$= 4 \cdot 3(x+1)^{1/3} \Big|_0^{26}$$

$$= 12\sqrt[3]{x+1} \Big|_0^{26}$$

$$= (\ 36\ ) - (\ 12\ )$$

$$= 24$$

$g(x) = 4(x+1)^{-2/3}$ and let f be the function defined by $f(x) = \int_0^x g(t)\,dt$ for $x \geq 0$.

(b) Determine the concavity of the graph of $y = f(x)$ for $x > 0$. Justify your answer.

$f'(x) = g(x)$

$f''(x) = g'(x) = 4\left(-\frac{2}{3}\right)(x+1)^{-5/3}(1)$

$= -\dfrac{8}{3\sqrt[3]{(x+1)^5}}$

For $x>0$, $f''(x)<0$ $\therefore$ f is concave down for $x>0$.

$g(x) = 4(x+1)^{-2/3}$ and let f be the function defined by $f(x) = \int_0^x g(t)\,dt$ for $x \ge 0$.

(c) Let h be the function defined by $h(x) = x - f(x)$. Find the minimum value of h on the interval $0 \le x \le 26$.

$h'(x) = 1 - f'(x)$

$= 1 - g(x)$

$= 1 - 4(x+1)^{-2/3}$

$h'(x) = 0 \rightarrow 1 = \dfrac{4}{\sqrt[3]{(x+1)^2}}$

$\sqrt[3]{(x+1)^2} = 4$

$(x+1)^2 = 64$

$x = 7$

$h(0) = 0$

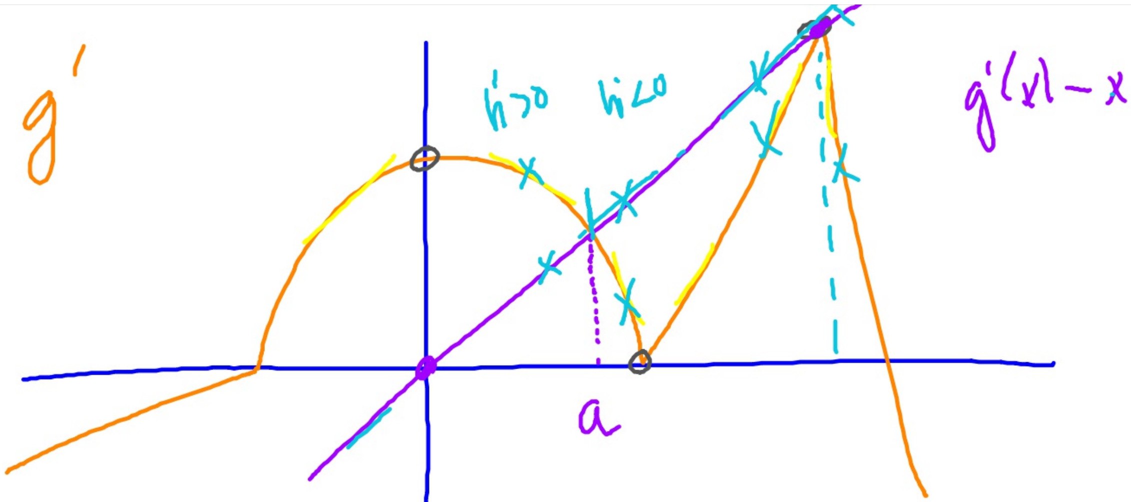
$h(26) = 26 - f(26) = 2$

$h(7) = 7 - \int_0^7 4(x+1)^{-2/3}\,dx$

$= 7 - \left[12\sqrt[3]{x+1}\right]_0^7$

$= -5.$

$=$



Possible IP at $x=0$, $x=2$ and $x=3$

g has IP at $x=0$ because $g''(x) > 0$ on $(-2,0)$ and $g''(x) < 0$ on $(0,2)$.

$$\int_0^3 g'(x)\,dx = g(3) - g(0) \qquad g(0) = 5 \quad g(3)$$

$$\# \quad = g(3) - 5$$

$$h(x) = g(x) - \frac{1}{2}x^2$$

$$h'(x) = g'(x) - x$$

$$h'(x) = 0 \rightarrow g'(x) = x$$

h has a rel. max at $x=a$

because on $(0,a)$ $g'(x)>x \rightarrow h'(x)>0$

and on $(a,2)$ $g'(x)<x \rightarrow h'(x)<0$.