18. For the function f, $f'(x) = 2x + 1$ and $f(1) = 4$. What is the approximation for $f(1.2)$ found by using the line tangent to the graph of f at $x = 1$?

(A) 0.6 (B) 3.4 (C) 4.2 (D) 4.6 (E) 4.64

$(1, 4)$

$f'(1) = 3$

$y - 4 = 3(x-1)$

$y = 4 + 3(x-1)$

$y(1.2) = 4 + 3(.2)$

$= 4.6$

19. Let f be the function given by $f(x) = x^3 - 6x^2$. The graph of f is concave up when

(A) $x > 2$

(B) $x < 2$

(C) $0 < x < 4$

(D) $x < 0$ or $x > 4$ only

(E) $x > 6$ only

$f'(x) = 3x^2 - 12x$

$f''(x) = 6x - 12$

$6x - 12 = 0 \Rightarrow x = 2$

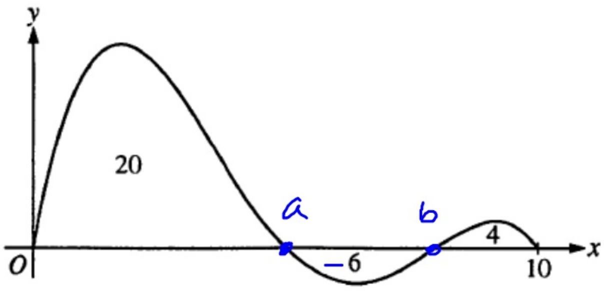
$-$ $+$

2

20. If $g(x) = x^2 - 3x + 4$ and $f(x) = g'(x)$, then $\int_1^3 f(x)\,dx =$

(A) $-\frac{14}{3}$ (B) -2 (C) 2 (D) 4 (E) $\frac{14}{3}$

$$\int_1^3 g'(x)\,dx = g(3) - g(1)$$
$$= 4 - 2$$
$$= 2$$



Graph of f'

EVT

21. The graph of f', the derivative of the function f, is shown above for $0 \le x \le 10$. The areas of the regions between the graph of f' and the x-axis are 20, 6, and 4, respectively. If $\boxed{f(0) = 2,}$ what is the maximum value of f on the closed interval $0 \le x \le 10$?

(A) 16 (B) 20 (C) 22 (D) 30 (E) 32

$f'(x) = 0 \rightarrow \boxed{x = a}$ or $x = b$

$f(0) = 2$

$f(10)$: $\int_0^{10} f'(x)\,dx = f(10) - f(0)$

$18 = f(10) - 2$

$f(10) = 20$

$f(a)$: $\int_0^{a} f'(x)\,dx = f(a) - f(0)$

$20 = f(a) - 2$

$22 = f(a)$

22. If $f'(x) = (x-2)\cancel{(x-3)}^2(x-4)^3$, then f has which of the following relative extrema?

I. A relative maximum at $x = 2$

II. A relative minimum at $x = 3$

III. A relative maximum at $x = 4$

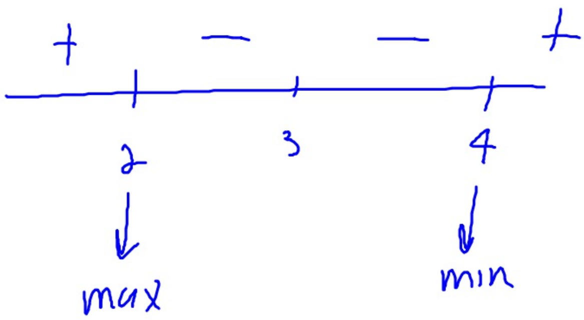
(A) I only

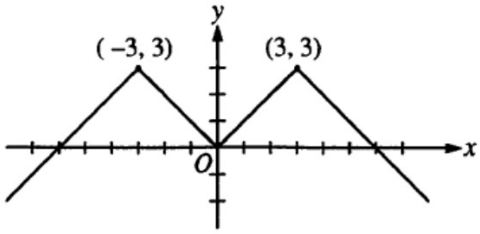
(B) III only

(C) I and III only

(D) II and III only

(E) I, II, and III





23. The graph of the even function $y = f(x)$ consists of 4 line segments, as shown above. Which of the following statements about f is false?

(A) $\lim_{x\to 0}(f(x) - f(0)) = 0$ ✓

(B) $\lim_{x\to 0}\dfrac{f(x) - f(0)}{x} = 0$ $\nexists$

(C) $\lim_{x\to 0}\dfrac{f(x) - f(-x)}{2x} = 0$

(D) $\lim_{x\to 2}\dfrac{f(x) - f(2)}{x - 2} = 1$

(E) $\lim_{x\to 3}\dfrac{f(x) - f(3)}{x - 3}$ does not exist.

24. The radius of a circle is increasing. At a certain instant, the rate of increase in the area of the circle is numerically equal to twice the rate of increase in its circumference. What is the radius of the circle at that instant?

(A) $\frac{1}{2}$ (B) 1 (C) $\sqrt{2}$ (D) 2 (E) 4

$\frac{dr}{dt} < 0$

$A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$C = 2\pi r$

$\frac{dC}{dt} = 2\pi \frac{dr}{dt}$

$\frac{dA}{dt} = 2\frac{dC}{dt}$

$2\cancel{\pi} r \cancel{\frac{dr}{dt}} = 4\cancel{\pi} \cancel{\frac{dr}{dt}}$

$2r = 4$

$r = 2$

25. If $x^2y - 3x = y^3 - 3$, then at the point $(-1, 2)$, $\frac{dy}{dx} =$

(A) $-\frac{7}{11}$ (B) $-\frac{7}{13}$ (C) $-\frac{1}{2}$ (D) $-\frac{3}{14}$ (E) 7

$$x^2\frac{dy}{dx} + 2xy - 3 = 3y^2\frac{dy}{dx}$$

$$\frac{dy}{dx} - 4 - 3 = 12\frac{dy}{dx}$$

$$-7 = 11\frac{dy}{dx}$$

$$\frac{-7}{11} = \frac{dy}{dx}$$

26. For $x > 0$, f is a function such that $f'(x) = \dfrac{\ln x}{x}$ and $f''(x) = \dfrac{1 - \ln x}{x^2}$. Which of the following is true?

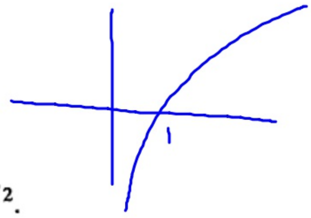
(A) f is decreasing for $x > 1$, and the graph of f is concave down for $x > e$.

(B) f is decreasing for $x > 1$, and the graph of f is concave up for $x > e$.

(C) f is increasing for $x > 1$, and the graph of f is concave down for $x > e$.

(D) f is increasing for $x > 1$, and the graph of f is concave up for $x > e$.

(E) f is increasing for $0 < x < e$, and the graph of f is concave down for $0 < x < e^{3/2}$.



$\ln x = 0$

$x = 1$

$-$ $+$

0 1

$\dfrac{\ln x}{x}$ $\dfrac{\ln \frac{1}{2}}{\frac{1}{2}}$ $-$ $+$

$\dfrac{\ln e}{e}$

$1 - \ln x = 0$

$\ln x = 1$

$x = e$

$+$ $-$

0 e

$\dfrac{1 - \ln x}{x^2}$

$\dfrac{1 - 2\ln e}{e^2}$

27. If f is the function given by $f(x) = \int_4^{2x} \sqrt{t^2 - t}\, dt$, then $f'(2) =$

(A) 0 (B) $\frac{7}{2\sqrt{12}}$ (C) $\sqrt{2}$ (D) $\sqrt{12}$ (E) $2\sqrt{12}$

$$\frac{d}{dx} \int_4^{2x} \sqrt{t^2 - t}$$

$$2\sqrt{4x^2 - 2x} - (\ )(0)$$

$$2\sqrt{12}$$

28. If $y = \sin^{-1}(5x)$, then $\frac{dy}{dx} =$

(A) $\frac{1}{1+25x^2}$

(B) $\frac{5}{1+25x^2}$

(C) $\frac{-5}{\sqrt{1-25x^2}}$

(D) $\frac{1}{\sqrt{1-25x^2}}$

(E) $\frac{5}{\sqrt{1-25x^2}}$

$$\frac{5}{\sqrt{1-25x^2}} \qquad \sin^{-1} \bigstar \rightarrow \frac{\bigstar'}{\sqrt{1-\bigstar^2}}$$