85. Let $y = f(x)$ define a twice-differentiable function and let $y = t(x)$ be the line tangent to the graph of f at $x = 2$. If $t(x) \geq f(x)$ for all real x, which of the following must be true?

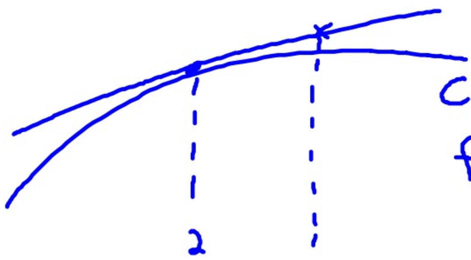
(A) $f(2) \geq 0$

(B) $f'(2) \geq 0$

(C) $f'(2) \leq 0$

(D) $f''(2) \geq 0$

(E) $f''(2) \leq 0$



CD

$f'' < 0$

86. The vertical line $x = 2$ is an asymptote for the graph of the function f. Which of the following statements must be false?

VA → $\lim_{x \to a} f(x) = \pm\infty$.

(A) $\lim_{x \to 2} f(x) = 0$

(B) $\lim_{x \to 2} f(x) = -\infty$

(C) $\lim_{x \to 2} f(x) = \infty$

(D) ~~$\lim_{x \to \infty} f(x) = 2$~~

(E) ~~$\lim_{x \to \infty} f(x) = \infty$~~

Graph ~~of f~~ h'

87. The graph of the piecewise linear function f is shown above. Let h be the function given by $\boxed{h(x) = \int_{-1}^{x} f(t)\,dt}$.

On which of the following intervals is h increasing? $\rightarrow h' > 0$ $\qquad h'(x) = f(x)$

(A) $[-1, 3]$

(B) $[0, 5]$

(C) $[2, 5]$ only

(D) $[2, 9]$

(E) $[3, 9]$ only

88. The first derivative of the function f is given by $f'(x) = \sin(x^2)$. At which of the following values of x does $\boxed{f \text{ have a local minimum?}}$ → Below to above

(A) 2.507 (B) 2.171 (C) 1.772 (D) 1.253 (E) 0

y1 = f'

Zoom box look for 1st time graph goes below to above

F5 zero to find it

89. If $\boxed{\lim_{x \to a} f(x) = f(a),}$ then which of the following statements about f must be true?

(A) f is continuous at $x = a$. f is cont at x=a

(B) f is differentiable at $x = a$.

(C) For all values of x, $f(x) = f(a)$.

(D) The line $y = f(a)$ is tangent to the graph of f at $x = a$.

(E) The line $x = a$ is a vertical asymptote of the graph of f.

90. The temperature F, in degrees Fahrenheit $(°F)$, of a cup of coffee t minutes after it is poured is given by $F(t) = 72 + 118e^{-0.093t}$. To the nearest degree, what is the average temperature of the coffee between $t = 0$ and $t = 10$ minutes?

$$\frac{1}{10}\int_0^{10} F(t)\,dt$$

(A) 93°F

(B) 119°F

(C) 146°F

(D) 149°F

(E) 154°F

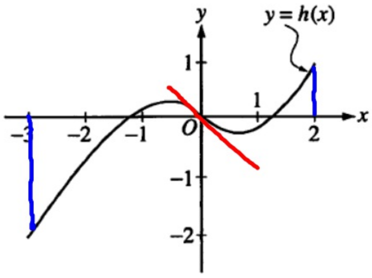
91. If $f'(x) = \cos(x^2)$ and $f(3) = 7$, then $f(2) =$

(A) 0.241 (B) 5.831 (C) 6.416 (D) 6.759 (E) 7.241

$$\int_a^b f'(x)\,dx = f(b) - f(a)$$

$$\int_2^3 \cos x^2\,dx = 7 - f(2)$$

$$f(2) = 7 - \int_2^3 \cos x^2\,dx$$



92. The graph of the function h is shown in the figure above. Of the following, which has the greatest value?

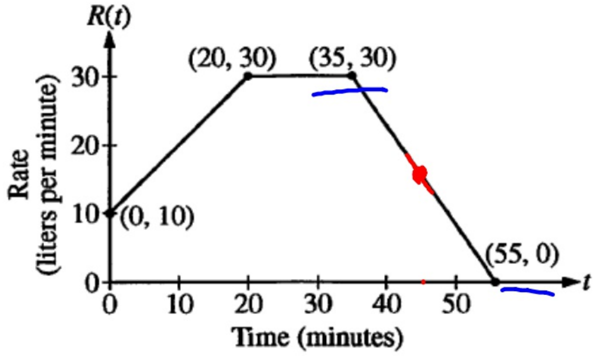
(A) Average value of h over $[-3,2]$ $\frac{1}{5}\int_{-3}^{2} h(x)\,dx < 0$

(B) Average rate of change of h over $[-3,2]$ $\frac{f(2)-f(-3)}{2--3} = \frac{1--2}{5} = \frac{3}{5}$

(C) $\int_{-3}^{2} h(x)\,dx$ < 0

(D) $\int_{-3}^{0} h(x)\,dx$ < 0

(E) $h'(0)$ < 0

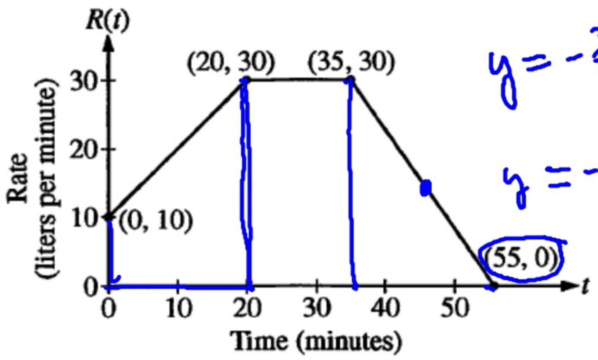


1. At time $t = 0$ minutes, a tank contains 100 liters of water. The piecewise-linear graph above shows the rate $R(t)$, in liters per minute, at which water is pumped into the tank during a 55-minute period.

(a) Find $R'(45)$. Using appropriate units, explain the meaning of your answer in the context of this problem.

$$R'(45) = \frac{30-0}{35-55} = -\frac{3}{2}$$

The rate at which water is pumped into tank is decreasing at $\frac{3}{2}$ L/min^2.



$y = -\frac{3}{2}(x-55)$

$y = -\frac{3}{2}(45-55)$

$= \left(-\frac{3}{2}\right)(-10)$

15

(b) How many liters of water have been pumped into the tank from time $t = 0$ to time $t = 55$ minutes? Show the work that leads to your answer.

$$\int_0^{55} R(t)\,dt = \frac{1}{2}(10+30)(20) + (15)(30) + \frac{1}{2}(20)(30)$$
$$= 1150$$

$\therefore$ 1150 liters.

(c) At time $t = 10$ minutes, water begins draining from the tank at a rate modeled by the function D, where $D(t) = 10e^{(\sin t)/10}$ liters per minute. Water continues to drain at this rate until time $t = 55$ minutes. How many liters of water are in the tank at time $t = 55$ minutes?

$$100 + 1150 - \int_{10}^{55} D(t)\,dt = 799.725$$

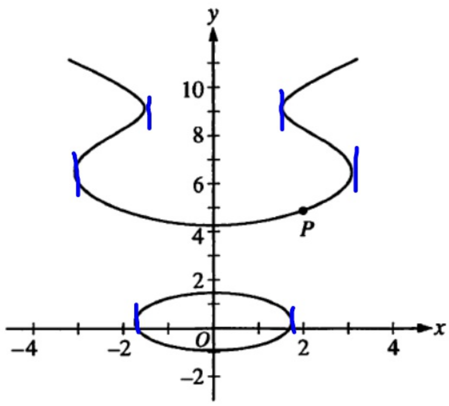
$\therefore$ 799.725 ~~liter~~ liter.

(d) Using the functions R and D, determine whether the amount of water in the tank is increasing or decreasing at time $t = 45$ minutes. Justify your answer.

$R(45) = 15$

$D(45) = 10.888$

The water level is increasing because $R(45) > D(45)$.



.201
~~2.440~~
6.485
9.223

2. The graph of the equation $x^2 = -2 + y + 5\cos y$ is shown above for $y \le 11$. It is known that $\frac{dy}{dx} = \frac{2x}{1 - 5\sin y}$.

The x-coordinate of point P shown on the graph is 2.

(a) Write an equation for the line tangent to the graph at point P.

$x = 2 \longrightarrow 4 = -2 + y + 5\cos y \Rightarrow y = 4.928$

$\left.\frac{dy}{dx}\right|_{(2,4.928)} = .680$

$\therefore\ y - 4.928 = .680(x-2).$

(b) For $y \leq 11$, find the y-coordinate of each point on the graph where the line tangent to the graph at that point is vertical.

$1 - 5\sin y = 0 \rightarrow y = .201$

or $y = 6.485$

or $y = 9.223$