

85. Let $y = f(x)$ define a twice-differentiable function and let $y = t(x)$ be the line tangent to the graph of f at $x = 2$. If $t(x) \geq f(x)$ for all real x , which of the following must be true?

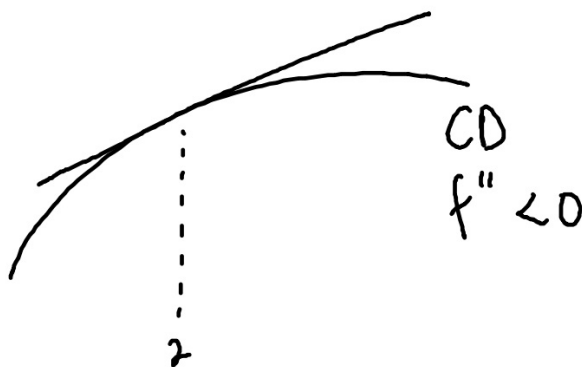
(A) $f(2) \geq 0$

(B) $f'(2) \geq 0$

(C) $f'(2) \leq 0$

(D) $f''(2) \geq 0$

(E) $f''(2) \leq 0$



86. The vertical line $x = 2$ is an asymptote for the graph of the function f . Which of the following statements must be false?

(A) $\lim_{x \rightarrow 2} f(x) = 0$

(B) $\lim_{x \rightarrow 2} f(x) = -\infty$

(C) $\lim_{x \rightarrow 2} f(x) = \infty$

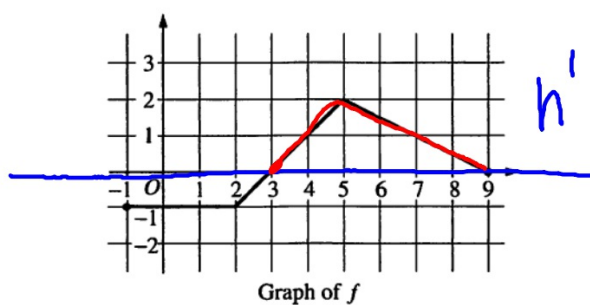
(D) $\lim_{x \rightarrow \infty} f(x) = 2$

(E) $\lim_{x \rightarrow \infty} f(x) = \infty$

VA

$$\lim_{x \rightarrow a} f(x) = \pm \infty.$$

~~2, 2~~
~~1, 1~~



87. The graph of the piecewise linear function f is shown above. Let h be the function given by $h(x) = \int_{-1}^x f(t) dt$.

On which of the following intervals is h increasing?

(A) $[-1, 3]$

(B) $[0, 5]$

(C) $[2, 5]$ only

(D) $[2, 9]$

(E) $[3, 9]$ only

$h' > 0$
 $(3, 9)$

$$h'(x) = f(x)$$

88. The first derivative of the function f is given by $f'(x) = \sin(x^2)$. At which of the following values of x does f have a local minimum?

BELOW TO ABOVE

- (A) 2.507 (B) 2.171 (C) 1.772 (D) 1.253 (E) 0

$y' = f'$ zero box

FS zero

1st time between 0 and 3
it goes below to above

89. If $\lim_{x \rightarrow a} f(x) = f(a)$ then which of the following statements about f must be true?

(A) f is continuous at $x = a$.

(B) f is differentiable at $x = a$.

(C) For all values of x , $f(x) = f(a)$.

(D) The line $y = f(a)$ is tangent to the graph of f at $x = a$.

(E) The line $x = a$ is a vertical asymptote of the graph of f .

f cont $x=a$

90. The temperature F , in degrees Fahrenheit ($^{\circ}\text{F}$), of a cup of coffee t minutes after it is poured is given by

$F(t) = 72 + 118e^{-0.093t}$. To the nearest degree, what is the average temperature of the coffee between $t = 0$ and $t = 10$ minutes?

(A) 93°F

(B) 119°F

(C) 146°F

(D) 149°F

(E) 154°F

$$\frac{1}{10} \int_0^{10} F(t) dt$$

91. If $f'(x) = \cos(x^2)$ and $f(3) = 7$, then $f(2) =$

(A) 0.241

(B) 5.831

(C) 6.416

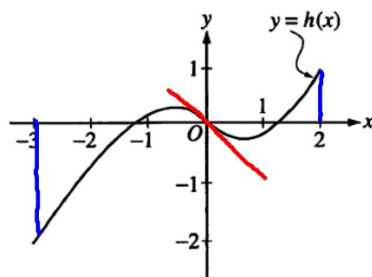
(D) 6.759

(E) 7.241

$$\int_a^b f'(x) dx = f(b) - f(a)$$

$$\int_2^3 \cos x^2 dx = 7 - f(2)$$

$$f(2) = 7 - \int_2^3 \cos x^2 dx$$



92. The graph of the function h is shown in the figure above. Of the following, which has the greatest value?

(A) Average value of h over $[-3, 2]$

(B) Average rate of change of h over $[-3, 2]$

(C) $\int_{-3}^2 h(x) dx$

(D) $\int_{-3}^0 h(x) dx$

(E) $h'(0)$

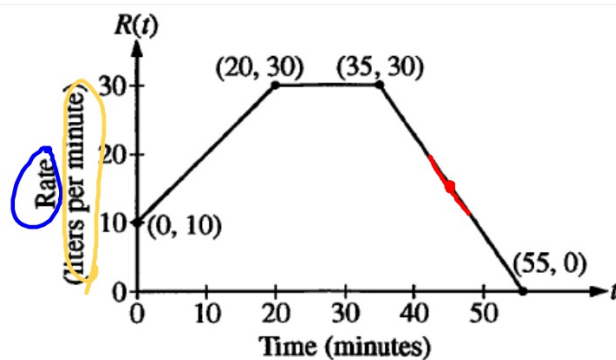
$$\frac{1}{5} \int_{-3}^2 h(x) dx < 0$$

$$\frac{h(2) - h(-3)}{5} = \frac{1 - (-2)}{5} = \frac{3}{5}$$

$$< 0$$

$$< 0$$

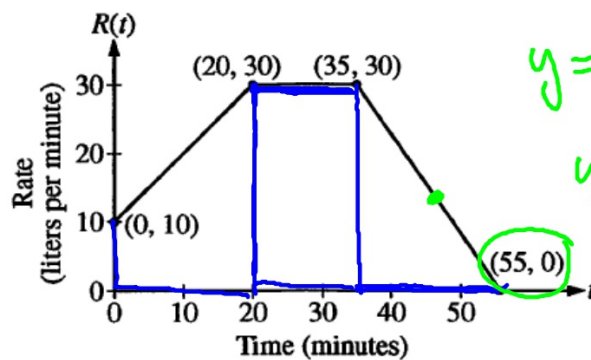
$$< 0$$



1. At time $t = 0$ minutes, a tank contains 100 liters of water. The piecewise-linear graph above shows the rate $R(t)$, in liters per minute, at which water is pumped into the tank during a 55-minute period.
- (a) Find $R'(45)$. Using appropriate units, explain the meaning of your answer in the context of this problem.

$$R'(45) = \frac{30 - 0}{35 - 55} = -\frac{3}{2}$$

The rate at which water is pumped into tank
is decreasing at $\frac{3}{2}$ l/min/min



- (b) How many liters of water have been pumped into the tank from time $t = 0$ to time $t = 55$ minutes? Show the work that leads to your answer.

$$\int_0^{55} R(t) dt = \frac{1}{2} (10 + 30)(20) + (30)(15) + \frac{1}{2} (20)(30) = 1150$$

$\therefore 1150$ liters.

- (c) At time $t = 10$ minutes, water begins draining from the tank at a rate modeled by the function D , where $D(t) = 10e^{(\sin t)/10}$ liters per minute. Water continues to drain at this rate until time $t = 55$ minutes. How many liters of water are in the tank at time $t = 55$ minutes?

$$100 + 1150 - \int_{10}^{55} D(t) dt = 799.725$$

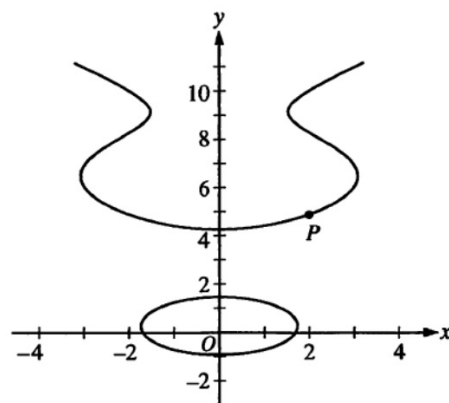
$\therefore 799.725$ liters.

- (d) Using the functions R and D , determine whether the amount of water in the tank is increasing or decreasing at time $t = 45$ minutes. Justify your answer.

$$R(45) = 15$$

$$D(45) = 10.888$$

The water level is increasing at $t = 45$ min
because $R(45) > D(45)$.



2. The graph of the equation $x^2 = -2 + y + 5 \cos y$ is shown above for $y \leq 11$. It is known that $\frac{dy}{dx} = \frac{2x}{1 - 5 \sin y}$.

The x -coordinate of point P shown on the graph is 2.

- (a) Write an equation for the line tangent to the graph at point P .

$$x=2 \rightarrow 4 = -2 + y + 5 \cos y \rightarrow y = 4.928$$

$$\left. \frac{dy}{dx} \right|_{(2, 4.928)} = .680$$

$$\therefore y - 4.928 = .680(x - 2).$$