1. $\int \cos(3x)\,dx =$

(A) $-3\sin(3x) + C$

(B) $-\frac{1}{3}\sin(3x) + C$

(C) $\frac{1}{3}\sin(3x) + C$

(D) $\sin(3x) + C$

(E) $3\sin(3x) + C$

$\frac{1}{3}\sin 3x + C$

2. $\lim_{x \to 0} \frac{2x^6 + 6x^3}{4x^5 + 3x^3}$ is

h

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) 2 (E) nonexistent

$\frac{0}{0}$ $\quad \frac{12x^5 + 18x^2}{20x^4 + 9x^2}$ $\quad \frac{0}{0}$ $\quad \frac{60x^4 + 36x}{80x^3 + 18x}$ $\quad \frac{0}{0}$

$\frac{240x + 36}{240x^2 + 18}$ $\quad \frac{36}{18} = 2$

$$f(x) = \begin{cases} x^2 - 3x + 9 & \text{for } x \le 2 \\ kx + 1 & \text{for } x > 2 \end{cases}$$

3. The function f is defined above. For what value of k, if any, is f continuous at $x = 2$?

(A) 1

(B) 2

(C) 3

(D) 7

(E) No value of k will make f continuous at $x = 2$.

$7 = 2k + 1$

$k = 3$

$f(a) = \lim_{x \to a} f(x)$

4. If $f(x) = \cos^3(4x)$, then $f'(x) =$

(A) $3\cos^2(4x)$

(B) $-12\cos^2(4x)\sin(4x)$

(C) $-3\cos^2(4x)\sin(4x)$

(D) $12\cos^2(4x)\sin(4x)$

(E) $-4\sin^3(4x)$

$$[3\cos^2 4x][-\sin 4x][4]$$

$$-12\cos^2 4x \sin 4x$$

5. The function f given by $f(x) = 2x^3 - 3x^2 - 12x$ has a relative minimum at $x =$

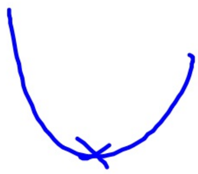
(A) -1 (B) 0 (C) 2 (D) $\frac{3 - \sqrt{105}}{4}$ (E) $\frac{3 + \sqrt{105}}{4}$

$f'(x) = 6x^2 - 6x - 12$

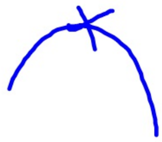
$x^2 - x - 2 = 0$

$(x - 2)(x + 1)$

$x = 2$ or $x = -1$



$f''(x) = 12x - 6$



$f''(2) = 24 - 6 > 0$ CU MIN

$f''(-1) = -12 - 6 < 0$ CD MAX

6. Let f be the function given by $f(x) = (2x-1)^5(x+1)$. Which of the following is an equation for the line tangent to the graph of f at the point where $x = 1$?

(A) $y = 21x + 2$

(B) $y = 21x - 19$

(C) $y = 11x - 9$

(D) $y = 10x + 2$

(E) $y = 10x - 8$

$f(1) = (1^5)(2) = 2$

$(1, 2)$

$f'(x) = (2x-1)^5 + (x+1)(5)(2x-1)^4(2)$

$f'(1) = 1 + (2)(5)(1)(2)$

$= 1 + 20 = 21$

$y - 2 = 21(x-1)$

$y - 2 = 21x - 21$

$y = 21x - 19$

7. $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx =$

(A) $2e^{\sqrt{x}} + C$

(B) $\frac{1}{2}e^{\sqrt{x}} + C$

(C) $e^{\sqrt{x}} + C$

(D) $2\sqrt{x}\,e^{\sqrt{x}} + C$

(E) $\frac{1}{2}\frac{e^{\sqrt{x}}}{\sqrt{x}} + C$

$u = \sqrt{x}$

$du = \frac{1}{2\sqrt{x}}\,dx$

$2\,du = \frac{1}{\sqrt{x}}\,dx$

$2\int e^{u}\,du$

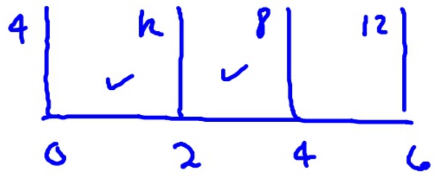
$2e^{u} + C$

$2e^{\sqrt{x}} + C$

x	0	2	4	6
$f(x)$	4	k	8	12

8. The function f is continuous on the closed interval $[0, 6]$ and has the values given in the table above. The trapezoidal approximation for $\int_0^6 f(x)\,dx$ found with 3 subintervals of equal length is 52. What is the value of k?

(A) 2 (B) 6 (C) 7 (D) 10 (E) 14



$$\frac{1}{2}(4+k)(2) + \frac{1}{2}(k+8)2 + \frac{1}{2}(20)(2)$$

$$4+k+k+8+20 = 52$$

$$2k+32=52$$

$$2k=20$$

$$k=10$$

9. A particle moves along the x-axis so that at any time $t > 0$, its velocity is given by $v(t) = 4 - 6t^2$. If the particle is at position $x = 7$ at time $t = 1$, what is the position of the particle at time $t = 2$?

(A) -10 (B) -5 (C) -3 (D) 3 (E) 17

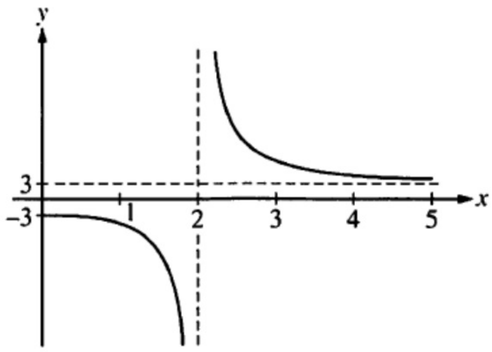
$$7 + \int_1^2 (4 - 6t^2)\,dt$$

$$7 + \left[4t - 2t^3\right]_1^2$$

$$7 + \left[(8 - 16) - (4 - 2)\right]$$

$$7 + [-8 - 2]$$

$$-3$$



HA $y=3$

$$\frac{a x^2+12}{1 x^2+b} = 3$$

10. The function f is given by $f(x) = \dfrac{ax^2 + 12}{x^2 + b}$. The figure above shows a portion of the graph of f. Which of the following could be the values of the constants a and b?

$a=3$

(A) $a = -3,\ b = 2$

(B) $a = 2,\ b = -3$

(C) $a = 2,\ b = -2$

(D) $a = 3,\ b = -4$

(E) $a = 3,\ b = 4$

$$\lim_{x \to 2} \frac{3x^2+12}{x^2+b}$$

$4+b=0$

$b=-4$

11. What is the slope of the line tangent to the graph of $y = \frac{e^{-x}}{x+1}$ at $x = 1$?

(A) $-\frac{1}{e}$ (B) $-\frac{3}{4e}$ (C) $-\frac{1}{4e}$ (D) $\frac{1}{4e}$ (E) $\frac{1}{e}$

$$\frac{dy}{dx} = \frac{(x+1)(-e^{-x}) - (e^{-x})}{(x+1)^2}$$

$$= \frac{-2e^{-1} - e^{-1}}{4}$$

$$= \frac{-3e^{-1}}{4} = -\frac{3}{4e}$$

12. If $f'(x) = \frac{2}{x}$ and $f(\sqrt{e}) = 5$, then $f(e) =$

(A) 2 (B) $\ln 25$ (C) $5 + \frac{2}{e} - \frac{2}{e^2}$ (D) 6 (E) 25

$$\int_a^b f'(x)\,dx = f(b) - f(a)$$

$2\ln\sqrt{e}$

$2\ln e^{1/2}$ ✓

$\frac{1}{2}\, 2\ln e$

$$\int_{\sqrt{e}}^{e} \frac{2}{x}\,dx = f(e) - f(\sqrt{e})$$

$1 = f(e) - 5$

$f(e) = 6$

$$2\int_{\sqrt{e}}^{e} \frac{1}{x}\,dx = 2\ln|x| \Big|_{\sqrt{e}}^{e}$$

$= (2\ln e) - (2\ln\sqrt{e})$

$= 2 - 1 = 1$

13. $\int (x^3+1)^2\,dx =$

(A) $\frac{1}{7}x^7 + x + C$

(B) $\frac{1}{7}x^7 + \frac{1}{2}x^4 + x + C$

(C) $6x^2(x^3+1) + C$

(D) $\frac{1}{3}(x^3+1)^3 + C$

(E) $\frac{(x^3+1)^3}{9x^2} + C$

$(x^3+1)(x^3+1)$

x^6+2x^3+1

$\int (x^6+2x^3+1)\,dx = \frac{1}{7}x^7 + \frac{2}{4}x^4 + x + C$

14. $\lim_{h\to 0} \frac{e^{(2+h)} - e^2}{h} =$

(A) 0 (B) 1 (C) $2e$ (D) e^2 (E) $2e^2$

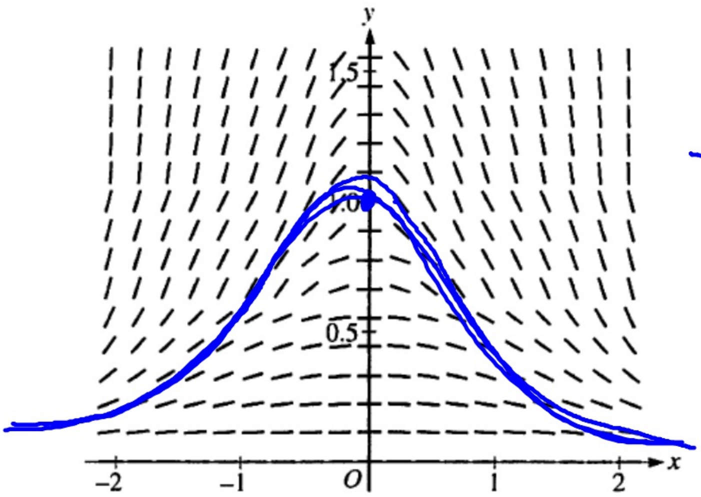
$f(x) = e^x$ $f'(2)$

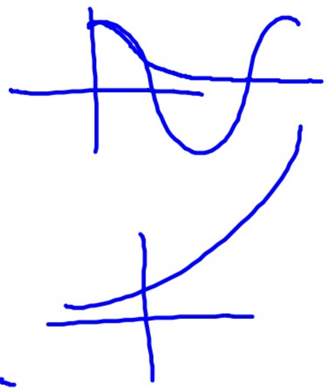
$f'(x) = e^x$

$f'(2) = e^2$

$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$

$f'(2)$





15. The slope field for a certain differential equation is shown above. Which of the following could be a solution to the differential equation with the initial condition $y(0) = 1$?

(A) $y = \cos x$

(B) $y = 1 - x^2$

(C) $y = e^x$

(D) $y = \sqrt{1 - x^2}$

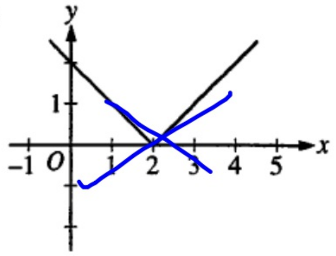
(E) $y = \dfrac{1}{1 + x^2}$

$y^2 = 1 - x^2$

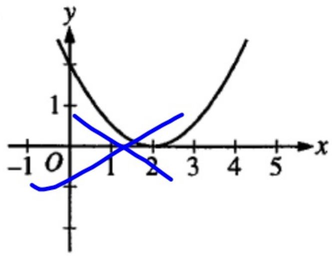
$x^2 + y^2 = 1$

16. If $f'(x) = |x-2|$, which of the following could be the graph of $y = f(x)$?

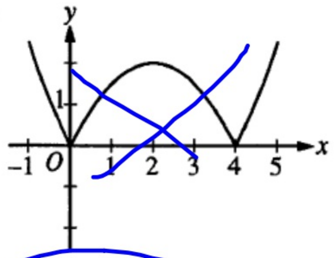
(A)



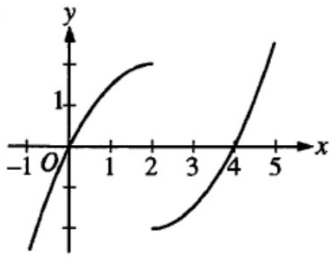
(B)



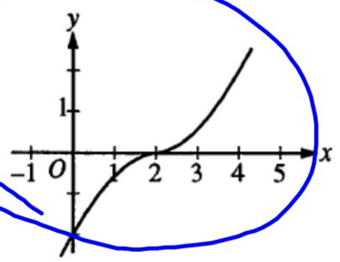
(C)



(D)



(E)



$f' > 0 \ \forall x \ \therefore f \uparrow$

$f'(2) = 0$ HT at $x = 2$

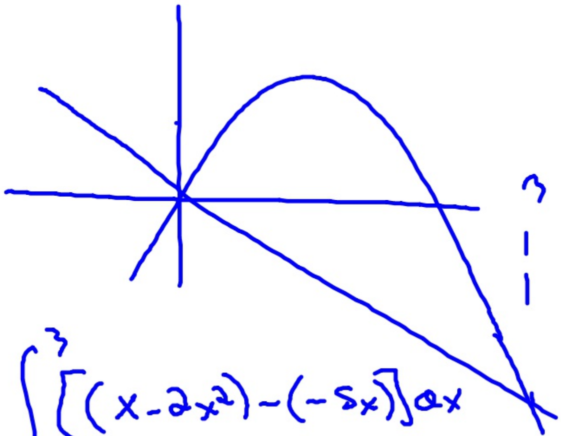
17. What is the area of the region enclosed by the graphs of $f(x) = x - 2x^2$ and $g(x) = -5x$?

(A) $\frac{7}{3}$ (B) $\frac{16}{3}$ (C) $\frac{20}{3}$ (D) 9 (E) 36

$x - 2x^2 = 0$

$x(1 - 2x) = 0$

$x = 0$ or $x = \frac{1}{2}$



$x - 2x^2 = -5x$

$0 = 2x^2 - 6x$

$0 = 2x(x - 3)$

$x = 0 \quad x = 3$

$\int_0^3 [(x - 2x^2) - (-5x)]\,dx$

$\int_0^3 [-2x^2 + 6x]\,dx$

$\left[-\frac{2}{3}x^3 + 3x^2\right]_0^3$

$(-18 + 27) - (0)$