11. $\lim_{x \to -3} \frac{x^2 - 9}{x^2 - 2x - 15}$ is

(A) 0 (B) $\frac{3}{5}$ (C) $\frac{3}{4}$ (D) 1 (E) nonexistent

$\frac{9-9}{9+6-15}$ $\frac{0}{0}$

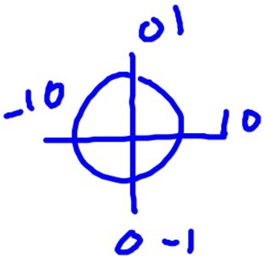
$\frac{2x}{2x-2}$ $\frac{-6}{-6-2}$ $\frac{3}{4}$

12. What is the average rate of change of $y = \cos(2x)$ on the interval $\left[0, \frac{\pi}{2}\right]$?

(A) $-\frac{4}{\pi}$ (B) -1 (C) 0 (D) $\frac{\sqrt{2}}{2}$ (E) $\frac{4}{\pi}$

$$\frac{f(b)-f(a)}{b-a}$$

$$\frac{\cos\left(2\,\frac{\pi}{2}\right)-\cos(2\cdot 0)}{\frac{\pi}{2}-0}$$



$$\frac{\cos\pi-\cos 0}{\frac{\pi}{2}}$$

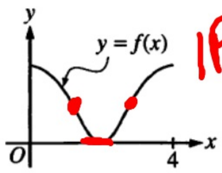
$$\frac{-1-1}{\frac{\pi}{2}} = -2\cdot\frac{2}{\pi}$$

13. If $y^3 + y = x^2$, then $\frac{dy}{dx} =$

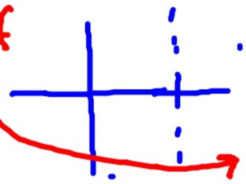
(A) 0 (B) $\frac{x}{2}$ (C) $\frac{2x}{3y^2}$ (D) $2x - 3y^2$ (E) $\frac{2x}{1+3y^2}$

$$3y^2 \frac{dy}{dx} + \frac{dy}{dx} = 2x$$

$$\frac{dy}{dx} = \frac{2x}{3y^2+1}$$



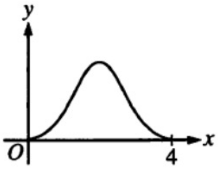
IP on f



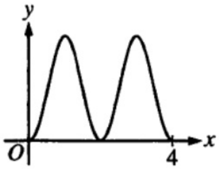
rel max/min on f'

14. The graph of $y = f(x)$ on the closed interval $[0, 4]$ is shown above. Which of the following could be the graph of $y = f'(x)$?

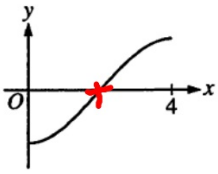
(A)



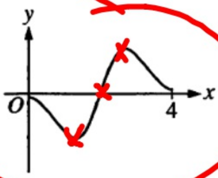
(B)



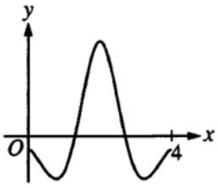
(C)



(D)



(E)



$$f(x)=\begin{cases}3x-2 & \text{if } x<1\\ \ln(3x-2) & \text{if } x\geq 1\end{cases}$$

15. Let f be the function defined above. Which of the following statements about f are true?

I. $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x)$ X

$x\to 1^-$ 1

II. $\lim_{x\to 1^-} f'(x) = \lim_{x\to 1^+} f'(x)$

$x\to 1^+$ 0

III. f is differentiable at $x=1$. X

(A) None

(B) I only

(C) II only

(D) II and III only

(E) I, II, and III

16. The function f is defined by $f(x) = 2x^3 - 4x^2 + 1$. The application of the Mean Value Theorem to f on the interval $1 \le x \le 3$ guarantees the existence of a value c, where $1 < c < 3$, such that $f'(c) =$

(A) 0 (B) 9 (C) 10 (D) 14 (E) 16

$$\boxed{f'(c) = \frac{f(b) - f(a)}{b - a}}$$

$$= \frac{f(3) - f(1)}{3 - 1}$$

$$= \frac{19 - -1}{2}$$

$$= 10$$

$2(27) - 4(9) + 1$

$54 - 36 + 1$

$2 - 4 + 1$

$-2 + 1$

17. The velocity v, in meters per second, of a certain type of wave is given by $v(h) = 3\sqrt{h}$, where h is the depth, in meters, of the water through which the wave moves. What is the rate of change, in meters per second per meter, of the velocity of the wave with respect to the depth of the water, when the depth is 2 meters?

(A) $-\frac{3}{4\sqrt{2}}$ (B) $-\frac{3}{8\sqrt{2}}$ (C) $\frac{3}{2\sqrt{2}}$ (D) $\frac{3}{\sqrt{2}}$ (E) $4\sqrt{2}$

$$v'(h) = 3\frac{1}{2\sqrt{h}}$$

$$v'(2) = \frac{3}{2\sqrt{2}}$$

18. If $\frac{dy}{dt} = -10e^{-t/2}$ and $y(0) = 20$, what is the value of $y(6)$?

(A) $20e^{-6}$ (B) $\boxed{20e^{-3}}$ (C) $20e^{-2}$ (D) $10e^{-3}$ (E) $5e^{-3}$

$$\int_a^b f'(x)\,dx = f(b) - f(a)$$

$$20e^{-3} - \cancel{20} = y(6) - \cancel{20}$$

$$-10\int_0^6 e^{-\frac{1}{2}t}\,dt = y(6) - 20$$

$$-10 \cdot -2e^{-\frac{1}{2}t}\Big|_0^6 = 20e^{-\frac{1}{2}t}\Big|_0^6 = 20e^{-3} - 20$$

19. Let f be the function with derivative defined by $f'(x) = x^3 - 4x$. At which of the following values of x does the graph of f have a point of inflection?

(A) 0 (B) $\frac{2}{3}$ (C) $\frac{2}{\sqrt{3}}$ (D) $\frac{4}{3}$ (E) 2

$f''(x) = 3x^2 - 4$

$3x^2 - 4 = 0$

$3x^2 = 4$

$x^2 = \frac{4}{3}$

$x = \pm \frac{2}{\sqrt{3}}$

20. Let f be the function given by $f(x) = \dfrac{(x-4)(2x-3)}{(x-1)^2}$. If the line $y = b$ is a horizontal asymptote to the graph of f, then $b =$

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

HA → lim ±∞

$$\frac{2x^2 + \cdots}{x^2 + \cdots}$$

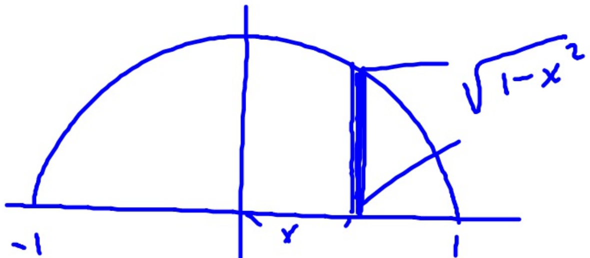
2

21. The base of a solid is the region bounded by the x-axis and the graph of $y = \sqrt{1 - x^2}$. For the solid, each cross section perpendicular to the x-axis is a square. What is the volume of the solid?

(A) $\frac{2}{3}$ (B) $\frac{4}{3}$ (C) 2 (D) $\frac{2\pi}{3}$ (E) $\frac{4\pi}{3}$

$y^2 = 1 - x^2$

$x^2 + y^2 = 1$



$$\int_{-1}^{1} \left(\sqrt{1-x^2}\right)^2 dx = 2\int_0^1 (1-x^2)\,dx = 2\left[x - \frac{1}{3}x^3\right]_0^1$$

$$= 2\left[\left(1 - \frac{1}{3}\right) - (\ 0\ )\right]$$

$$= (2)\left(\frac{2}{3}\right)$$

22. Let f be the function given by $f(x) = \dfrac{kx}{x^2+1}$, where k is a constant. For what values of k, if any, is f strictly decreasing on the interval $(-1, 1)$?

(A) $k < 0$

(B) $k = 0$

(C) $k > 0$

(D) $k > 1$ only

(E) There are no such values of k.

$1-(-2)^2$

$1-4$

$k(x^2+1-2x^2)$

$k(1-x^2)$

$$f'(x) = \frac{(x^2+1)(k)-(kx)(2x)}{(x^2+1)^2} = \frac{k(1-x^2)}{(x^2+1)^2}$$

$-$ $+$

$k(1-x^2)=0$

$k(1-x)(1+x)=0$

$k=0$ $k=1$ a $k=-1$

$-$ $+$ $-$

-1 1

$k<0$

23. Which of the following is an equation for the line tangent to the graph of $y = 3 - \int_{-1}^{x} e^{-t^3}\,dt$ at the point where $x = -1$?

(A) $y - 3 = -3e(x+1)$

(B) $y - 3 = -e(x+1)$

(C) $y - 3 = 0$

(D) $y - 3 = \frac{1}{e}(x+1)$

(E) $y - 3 = 3e(x+1)$

$$\frac{dy}{dx} = -e^{-x^3}$$

$$\left.\frac{dy}{dx}\right|_{x=-1} = -e^{-(-1)^3} = -e$$

$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$

24. Which of the following is the solution to the differential equation $\frac{dy}{dx} = 5y^2$ with the initial condition $y(0) = 3$?

(A) $y = \sqrt{9e^{5x}}$

(B) $y = \sqrt{\frac{1}{9}e^{5x}}$

(C) $y = \sqrt{e^{5x} + 9}$

(D) $y = \frac{3}{1 - 15x}$

(E) $y = \frac{3}{1 + 15x}$

$$\frac{dy}{dx} = 5y^2$$

$$y^{-2}\,dy = 5\,dx$$

$$-y^{-1} = 5x + C$$

$$-\frac{1}{y} = 5x + C$$

$$-\frac{1}{3} = C$$

$$-\frac{1}{y} = 5x - \frac{1}{3}$$

$$\frac{1}{y} = \frac{1}{3} - 5x$$

$$\frac{1}{y} = \frac{1 - 15x}{3}$$

$$y = \frac{3}{1 - 15x}$$

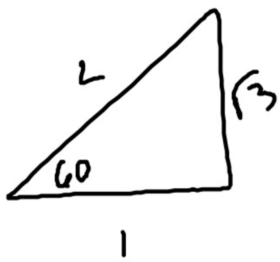
25. $\lim_{h \to 0} \frac{\sin\left(\frac{\pi}{3}+h\right)-\sin\left(\frac{\pi}{3}\right)}{h}$ is

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) $\frac{\sqrt{3}}{2}$ (E) nonexistent

$f(x) = \sin x$ $f'\left(\frac{\pi}{3}\right)$

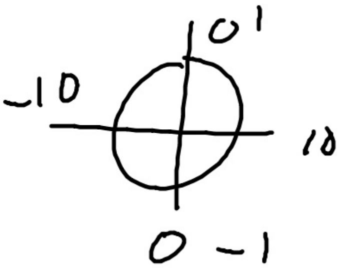
$f'(x) = \cos x$

$f'\left(\frac{\pi}{3}\right) = \cos\frac{\pi}{3} = \frac{1}{2}$



26. An object moves along a straight line so that at any time $t \geq 0$ its velocity is given by $v(t) = 2\cos(3t)$. What is the distance traveled by the object from $t = 0$ to the first time that it stops?

(A) 0 (B) $\frac{\pi}{6}$ (C) $\frac{2}{3}$ (D) $\frac{\pi}{3}$ (E) $\frac{4}{3}$



$$2\cos 3t = 0$$

$$\cos 3t = 0$$

$$3t = \frac{\pi}{2}$$

$$t = \frac{\pi}{6}$$

$$2\int_0^{\frac{\pi}{6}} \cos 3t \, dt = \frac{(2)}{3} \sin 3t \Big|_0^{\frac{\pi}{6}} = \left(\frac{2}{3}\sin\frac{\pi}{2}\right) - \left(\frac{2}{3}\sin 0\right)$$

$$= \frac{2}{3} - 0$$

x	$f(x)$	$f'(x)$
0	49	0
1	2	-8
2	-1	-80

C

27. The table above gives selected values for a differentiable and decreasing function f and its derivative. If f^{-1} is the inverse function of f, what is the value of $\left(f^{-1}\right)'(2)$?

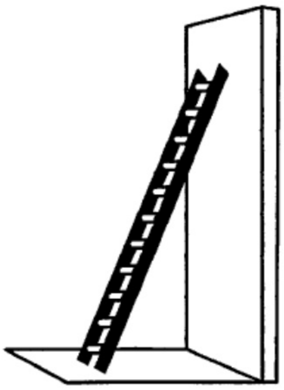
(A) -80 (B) $-\frac{1}{8}$ (C) $-\frac{1}{80}$ (D) $\frac{1}{80}$ (E) $\frac{1}{8}$

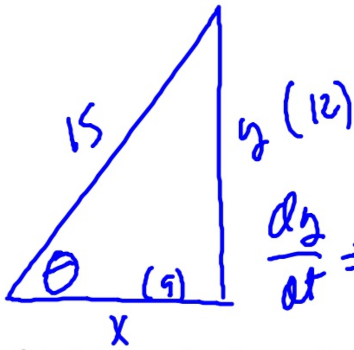
If (c,d) is on f $\rightarrow$ $(f^{-1})'(d) = \dfrac{1}{f'(c)} = \dfrac{1}{f'(1)}$

$= \dfrac{1}{-8}$

Find $\frac{d\theta}{dt}$

$\cos\theta = \frac{9}{15}$





$\frac{dy}{dt} = -2$

28. The top of a 15-foot-long ladder rests against a vertical wall with the bottom of the ladder on level ground, as shown above. The ladder is sliding down the wall at a constant rate of 2 feet per second. At what rate, in radians per second, is the acute angle between the bottom of the ladder and the ground changing at the instant the bottom of the ladder is 9 feet from the base of the wall?

(A) $-\frac{2}{9}$ (B) $-\frac{1}{6}$ (C) $-\frac{2}{25}$ (D) $\frac{2}{25}$ (E) $\frac{1}{9}$

$\sin\theta = \frac{1}{15}y$

$\cos\theta \frac{d\theta}{dt} = \frac{1}{15}\frac{dy}{dt}$

$\frac{d\theta}{dt} = \left[\frac{1}{15}(-2)\right] / \cos\theta$

$-\frac{2}{15} \cdot \frac{15}{9}$

$-\frac{2}{9}$

76. The function $P(t)$ models the population of the world, in billions of people, where t is the number of years since January 1, 2010. Which of the following is the best interpretation of the statement $P'(1) = 0.076$?

(A) On February 1, 2010, the population of the world was increasing at a rate of 0.076 billion people per year.

(B) On January 1, 2011, the population of the world was increasing at a rate of 0.076 billion people per year.

(C) On January 1, 2011, the population of the world was 0.076 billion pcoplc.

(D) From January 1, 2010 to January 1, 2011, the population of the world was increasing at an average rate of 0.076 billion people per year.

(E) When the population of the world was 1 billion people, the population of the world was increasing at a rate of 0.076 billion people per year.

x	0	2	4	6	8	10
$f(x)$	5	7	8	0	–15	–20

77. Let f be a differentiable function with selected values given in the table above. What is the average rate of change of f over the closed interval $0 \le x \le 10$?

(A) -6 (B) $-\frac{5}{2}$ (C) -2 (D) $-\frac{2}{5}$ (E) $\frac{2}{5}$

Avg rate $\frac{f(b) - f(a)}{b - a}$

$$\frac{-20 - 5}{10 - 0}$$

$$-\frac{25}{10}$$

$$-\frac{5}{2}$$

78. The rate at which motor oil is leaking from an automobile is modeled by the function L defined by $L(t) = 1 + \sin(t^2)$ for time $t \geq 0$. $L(t)$ is measured in liters per hour, and t is measured in hours. How much oil leaks out of the automobile during the first half hour?

(A) 1.998 liters

(B) 1.247 liters

(C) 0.969 liters

(D) 0.541 liters

(E) 0.531 liters

$$\int_0^{1/2} L(t)\,dt$$

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
0	3	4	2	π

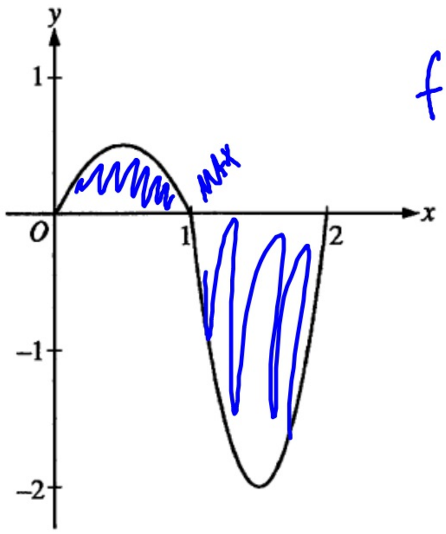
79. The table above gives values of the differentiable functions f and g and their derivatives at $x = 0$. If $h(x) = \frac{f(x)}{g(x)}$, what is the value of $h'(0)$?

(A) $\frac{8 - 3\pi}{4}$ (B) $\frac{3\pi - 8}{4}$ (C) $\frac{4}{\pi}$ (D) $\frac{2 - 3\pi}{2}$ (E) $\frac{8 + 3\pi}{4}$

$$h'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$$

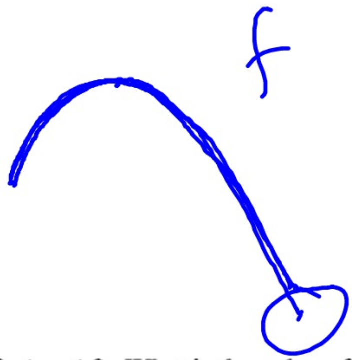
$$h'(0) = \frac{g(0)f'(0) - f(0)g'(0)}{[g(0)]^2}$$

$$= \frac{(2)(4) - (3)\pi}{4} \qquad \frac{8 - 3\pi}{4}$$



Graph of f'

f decr more on (1,2) than it incr. on (0,1)



80. The figure above shows the graph of f', the derivative of a function f, for $0 \leq x \leq 2$. What is the value of x at which the absolute minimum of f occurs?

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) $\frac{3}{2}$ (E) 2

81. What is the area of the region enclosed by the graphs of $y = \sqrt{4x - x^2}$ and $y = \frac{x}{2}$?

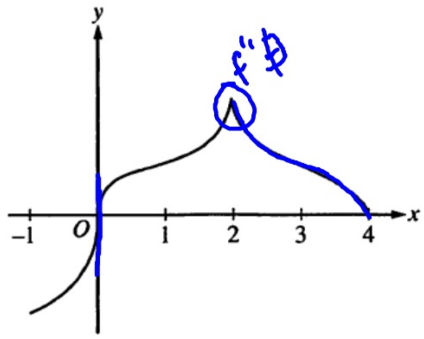
(A) 1.707 (B) 2.829 (C) 5.389 (D) 8.886 (E) 21.447

$$\int_{0}^{3.200}$$

$\int(y1(x) - y2(x), x, 0, a) = 2.829$

FS

$xc \Rightarrow a$



Graph of f'

f' at $x=2$
rel ext.
then f has IP at $x=2$

82. The graph of f', the derivative of f, is shown above. The line tangent to the graph of f' at $x = 0$ is vertical, and f' is not differentiable at $x = 2$. Which of the following statements is true?

(A) f' does not exist at $x = 2$. F

(B) f is decreasing on the interval $(2, 4)$. F

(C) The graph of f has a point of inflection at $x = 2$.

(D) The graph of f has a point of inflection at $x = 0$.

(E) f has a local maximum at $x = 0$.

83. A particle moves along the x-axis so that its position at time $t > 0$ is given by $x(t)$ and

V $\frac{dx}{dt} = -10t^4 + 9t^2 + 8t$. The acceleration of the particle is zero when $t =$ y1

(A) 0.387 (B) 0.831 (C) 1.243 (D) 1.647 (E) 8.094

y2 = d(y1(x), x).

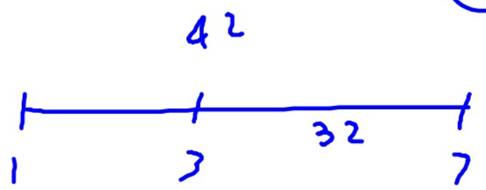
a(t) is y2(x) !

zeros(y2(x), x) → .831

84. The function f is continuous on the closed interval $[1, 7]$. If $\int_1^7 f(x)\,dx = 42$ and $\int_7^3 f(x)\,dx = -32$, then $\int_1^3 2f(x)\,dx =$

(A) -148 (B) 10 (C) 12 (D) 20 (E) 148

$\int_3^7 = 32$



$\int_1^3 = 10$

$2\int_1^3 f(x)\,dx = 2(10) = \underline{20}$