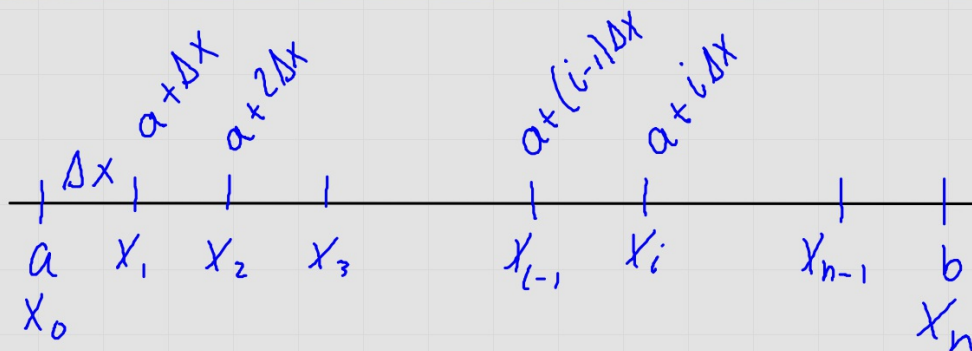


Exact Area via the Limit of a Riemann Sum



$$A \approx \sum_{i=1}^n f(x_i) \Delta x$$

$$A \approx \sum_{i=0}^{n-1} f(x_i) \Delta x$$

$$\approx \sum_{i=1}^n f(x_{i-1}) \Delta x$$

$$A \approx \sum_{i=1}^n f(c_i) \Delta x$$

$$c_i = x_i = a + i \Delta x \quad \text{RS}$$

$$c_i = x_{i-1} = a + (i-1) \Delta x \quad \text{LS}$$

$$A = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(c_i) \Delta x$$

$$A = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i) \Delta x$$

Bands: $f(x) = 5x + 1$, $x=2$, $x=6$, x -axis

$$\Delta x = \frac{4}{n} \quad x_i = 2 + i\Delta x$$

$$A = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(2 + i\Delta x) \Delta x$$

$$= \lim_{n \rightarrow \infty} \sum_{i=1}^n [11\Delta x + 5i(\Delta x)^2]$$

$$= \lim_{n \rightarrow \infty} \sum_{i=1}^n \left[\frac{44}{n} + \frac{80}{n^2} i \right]$$

$$= \lim_{n \rightarrow \infty} \left[44 + \frac{80}{n^2} \frac{n^2 + n}{2} \right]$$

$$= 44 + 40(1)$$

$$= 84$$

Bounds: $f(x) = x^2$, $x=1$, $x=3$, x -axis

$$\Delta x = \frac{2}{n} \quad x_i = 1 + i\Delta x$$

$$A = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(1 + i\Delta x) \Delta x$$

$$= \lim_{n \rightarrow \infty} \sum_{i=1}^n \left[\Delta x + 2i(\Delta x)^2 + i^2(\Delta x)^3 \right]$$

$$= \lim_{n \rightarrow \infty} \sum_{i=1}^n \left[\frac{2}{n} + \frac{8}{n^2}i + \frac{8}{n^3}i^2 \right]$$

$$= \lim_{n \rightarrow \infty} \left[2 + \frac{8}{n^2} \frac{n^2+n}{2} + \frac{8}{n^3} \frac{2n^3+3n^2+n}{6} \right]$$

$$= 2 + 4(1) + \frac{4}{3}(2)$$

$$= \frac{26}{3}$$

Bounds: $f(x) = x^2 + x$, $x=2$, $x=5$, x -axis.

$$\Delta x = \frac{3}{n} \quad x_i = 2 + i\Delta x$$

$$A = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(2 + i\Delta x) \Delta x$$

$$= \lim_{n \rightarrow \infty} \sum_{i=1}^n \left[6\Delta x + 5i(\Delta x)^2 + i^2(\Delta x)^3 \right]$$

$$= \lim_{n \rightarrow \infty} \sum_{i=1}^n \left[\frac{18}{n} + \frac{45}{n^2} i + \frac{27}{n^3} i^2 \right]$$

$$= \lim_{n \rightarrow \infty} \left[18 + \frac{45}{n^2} \frac{n^2+1}{2} + \frac{27}{n^3} \frac{2n^3+3n^2+n}{6} \right]$$

$$= 18 + \frac{45}{2}(1) + \frac{9}{2}(2)$$

$$= \frac{99}{2}$$