Differential Equations--Day 2

$\frac{dy}{dx} = e^y(3x^2 - 6x)$ $(1,0)$ on curve find $y = f(x)$.

$e^{-y}\,dy = (3x^2 - 6x)\,dx$

$-e^{-y} = x^3 - 3x^2 + C$

$-e^0 = 1 - 3 + C$

$-1 = -2 + C$

$C = 1$

$-e^{-y} = x^3 - 3x^2 + 1$

$e^{-y} = -x^3 + 3x^2 - 1$

$-y = \ln(-x^3 + 3x^2 - 1)$

$y = -\ln(-x^3 + 3x^2 - 1)$

$\frac{dy}{dx} = y\cos x \quad y = 3 \text{ when } x = 0 \quad \text{find } y = f(x).$

$\frac{1}{y}\,dy = \cos x\,dx$

$\ln|y| = \sin x + C$

$y = e^{\sin x + C}$

$y = Ae^{\sin x}$

$3 = Ae^{\sin 0}$

$3 = Ae^{0} \rightarrow A = 3$

$y = 3e^{\sin x}$

$\frac{dy}{dx} = \frac{1+x}{xy}$ $\quad y(1) = -4$ find $y = f(x)$.

$y\,dy = \frac{1+x}{x}\,dx$

$\frac{1}{2}y^2 = \ln|x| + x + C$

$8 = \ln 1 + 1 + C$

$7 = C$

$\frac{1}{2}y^2 = \ln|x| + x + 7$

$y^2 = 2\ln|x| + 2x + 14$

$y = \pm\sqrt{2\ln|x| + 2x + 14}$

Since $y(1) = -4$,

$y = -\sqrt{2\ln|x| + 2x + 14}$

Aside

$\int \left(\frac{1}{x} + 1\right) dx$

$= \ln|x| + x + C$

$$\frac{dB}{dt} = \frac{1}{5}(100 - B) \quad \text{if } B(0) = 20 \text{ find } B(t)$$

$$\frac{1}{100 - B}\, dB = \frac{1}{5}\, dt$$

$$-\ln|100 - B| = \frac{1}{5}t + C$$

$$\ln|100 - B| = -\frac{1}{5}t + D$$

$$100 - B = e^{-\frac{1}{5}t + D}$$

$$100 - B = Ae^{-\frac{1}{5}t}$$

$$B = 100 - Ae^{-\frac{1}{5}t}$$

$$20 = 100 - Ae^{0}$$

$$A = 80$$

$$B(t) = 100 - 80e^{-\frac{1}{5}t}$$