Differential Equations

"find $y = f(x)$"

$$\frac{dy}{dx} = \frac{2x^2}{3y^3}$$

$$\int 3y^3\,dy = \int 2x^2\,dx$$

$$\frac{1}{4}y^4 = \frac{1}{3}x^3 + C$$

$$3y^4 = 4x^3 + D$$

$$\frac{d^2y}{dx^2} = 4x - 3$$

$$\frac{dy'}{dx} = 4x - 3$$

$$\int dy' = \int (4x - 3)\, dx$$

$$\frac{dy}{dx} = 2x^2 - 3x + C$$

$$\int dy = \int (2x^2 - 3x + C)\, dx$$

$$y = \frac{2}{3}x^3 - \frac{3}{2}x^2 + Cx + D$$

$$\frac{dy}{dx} = \frac{\ln x}{xy + xy^3}$$

$$\frac{dy}{dx} = \frac{\ln x}{x(y + y^3)}$$

$$\int (y + y^3)\,dy = \int \frac{\ln x}{x}\,dx$$

$$\frac{1}{2}y^2 + \frac{1}{4}y^4 = \frac{1}{2}\ln^2 x + C$$

$$2y^2 + y^4 = 2\ln^2 x + D$$

Aside

$$\int \frac{\ln x}{x}\,dx$$

$u = \ln x$

$du = \frac{1}{x}\,dx$

$$\int u\,du = \frac{1}{2}u^2 + C$$

$$= \frac{1}{2}(\ln x)^2 + C$$

$\frac{dy}{dx} = \frac{3x^2}{e^{2y}}$ $\quad f(0) = \frac{1}{2}$ find $y = f(x)$.

$\int e^{2y}\,dy = \int 3x^2\,dx$

$\frac{1}{2}e^{2y} = x^3 + C$

$\frac{1}{2}e = C$

$\frac{1}{2}e^{2y} = x^3 + \frac{1}{2}e$

$e^{2y} = 2x^3 + e$

$2y = \ln(2x^3 + e)$

$y = \frac{1}{2}\ln(2x^3 + e)$

or

$y = \ln\sqrt{2x^3 + e}$

$\frac{dy}{dx} = \frac{x}{y}$ $\quad y(3) = 4$ $\quad$ find $y = f(x)$.

$$y\,dy = x\,dx$$

$$\frac{1}{2}y^2 = \frac{1}{2}x^2 + C$$

$$8 = \frac{9}{2} + C$$

$$\frac{7}{2} = C$$

$$\frac{1}{2}y^2 = \frac{1}{2}x^2 + \frac{7}{2}$$

$$y^2 = x^2 + 7$$

$$y = \pm\sqrt{x^2+7}$$

Since $f(3) = 4$

$$y = \sqrt{x^2+7}$$