

More on Antidifferentiation and Substitution

$$\begin{aligned} & \int (x^2 - 6x + 9)^{1/3} dx \\ &= \int [(x-3)^2]^{1/3} dx \\ &= \int (x-3)^{2/3} dx \\ &= \frac{3}{5} (x-3)^{5/3} + C \end{aligned}$$

$$\int \frac{x^2 + 5x + 6}{x+3} dx = \int (x+2) dx$$

Rational Funct $\rightarrow$ 'N is bigger 'D by 1.

$$\int \frac{x^2 + 5x - 3}{x - 4} dx$$
$$= \int \left(x + 9 + \frac{33}{x-4} \right) dx$$

$$= \frac{1}{2}x^2 + 9x + 33 \ln|x-4| + C$$

$$\begin{array}{r} 4 \overline{) \begin{array}{rrr} 1 & 5 & -3 \\ & 4 & 36 \\ \hline 1 & 9 & 33 \end{array}} \\ x + 9 + \frac{33}{x-4} \end{array}$$

$$\int \left(x + 3 - \frac{15}{2x+1} \right) dx$$

$$= \frac{1}{2}x^2 + 3x - \frac{15}{2} \ln|2x+1| + C$$

$$\int \tan x \, dx = \int \frac{\sin x}{\cos x} \, dx$$

$$2 \ln x = \ln x^2$$

$$\begin{array}{l} \cancel{u = \sin x} \\ \cancel{du = \cos x \, dx} \end{array}$$

$$\begin{array}{l} u = \cos x \\ du = -\sin x \, dx \\ -du = \sin x \, dx \end{array}$$

$$= - \int \frac{1}{u} \, du$$

$$= - \ln|u| + C$$

$$= \ln|u'| + C = \ln|\sec x| + C$$

$$\int \tan u \, du = \ln|\sec u| + C$$

$$\int \cot x \, dx = \int \frac{\cos x}{\sin x} \, dx$$

$$u = \sin x$$

$$du = \cos x \, dx$$

$$= \int \frac{1}{u} \, du = \ln|u| + C$$

$$= \ln|\sin x| + C$$

$$\int \cot u \, du = \ln|\sin u| + C \quad \text{--- } \frac{1}{\cos u}$$

$$\int \sec x \, dx = \int \frac{\sec x (\sec x + \tan x)}{\sec x + \tan x} \, dx$$

$$u = \sec x + \tan x$$

$$du = (\sec x \tan x + \sec^2 x) \, dx$$

$$= \int \frac{1}{u} \, du = \ln|u| + C$$

$$= \ln|\sec x + \tan x| + C$$

$$\int \sec u \, du = \ln|\sec u + \tan u| + C$$

$$\int \csc u \, du = \ln|\csc u - \cot u| + C.$$

$$\int 3x \tan x^2 dx$$

$$u = x^2$$

$$du = 2x dx$$

$$\frac{1}{2} du = x dx$$

$$\frac{3}{2} \int \tan u du$$

$$= \frac{3}{2} \ln |\sec u| + C$$

$$= \frac{3}{2} \ln |\sec x^2| + C.$$

$$\int \cot(8x-3) dx$$

$$= \frac{1}{8} \ln |\sin(8x-3)| + C$$