# Antidifferentiation via Substitution

$$\int u^n \, du = \frac{u^{n+1}}{n+1} + C \qquad n \neq -1$$

$$\int \frac{1}{u} \, du = \ln|u| + C$$

$$\int \sin u \, du = -\cos u + C \qquad \int e^u \, du = e^u + C$$

$$\int \cos u \, du = \sin u + C \qquad \int a^u \, du = \frac{a^u}{\ln a} + C$$

$$\int \frac{1}{\sqrt{a^2-u^2}}\,du = \sin^{-1}\frac{u}{a}+C$$

$$\int \frac{1}{a^2+u^2}\,du = \frac{1}{a}\tan^{-1}\frac{u}{a}+C$$

$$\int \frac{1}{u\sqrt{u^2-a^2}}\,du = \frac{1}{a}\sec^{-1}\frac{u}{a}+C.$$

$$\int (3x+4)^5\,dx = \frac{1}{3}\int u^5\,du \qquad = \frac{1}{3}\frac{(3x+4)^6}{6}$$

$u = 3x+4$

$du = 3\,dx$

$\frac{1}{3}du = dx$

$$= \frac{1}{3}\,\frac{1}{6}u^6 + C$$

$$= \frac{1}{18}(3x+4)^6 + C.$$

$$\int e^{2x+3}\,dx = \frac{1}{2}\int e^u\,du$$

$u = 2x+3$

$du = 2\,dx$

$\frac{1}{2}du = dx$

$$= \frac{1}{2}e^u + C = \frac{1}{2}e^{2x+3} + C.$$

$$\int x \cos x^2 \, dx = \frac{1}{2} \int \cos u \, du$$

$$u = x^2$$

$$du = 2x \, dx$$

$$\frac{1}{2} du = x \, dx$$

$$= \frac{1}{2} \sin u + C$$

$$= \frac{1}{2} \sin x^2 + C.$$

$$\int 2x \cos x^2 \, dx = \int \cos u \, du$$

$$u = x^2$$

$$du = 2x \, dx$$

$$= \sin u + C$$

$$= \sin x^2 + C.$$

$$\int 6x \cos x^2 \, dx = 3\int \cos u \, du$$

$u = x^2$

$du = 2x \, dx$

$3\,du = 6x \, dx$

$$= 3 \sin u + C$$

$$= 3 \sin x^2 + C$$

$$\int 3\underline{e^x}\, 5^{e^x}\, \underline{dx} = 3\int 5^u\, du$$

$u = e^x$

$du = e^x\, dx$

$$= 3\,\frac{5^u}{\ln 5} + C$$

$$= \frac{3\left(5^{e^x}\right)}{\ln 5} + C.$$

$$\int \frac{x}{1+x^2}\,dx = \frac{1}{2}\int \frac{1}{u}\,du$$

$u = 1+x^2$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$$= \frac{1}{2}\ln|u| + C$$

$$= \frac{1}{2}\ln|1+x^2| + C.$$

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$$\int \frac{x\,dx}{(1+x^2)^3} = \frac{1}{2}\int u^{-3}\,du$$

$u = 1+x^2$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$$= \frac{1}{2}\frac{u^{-2}}{-2} + C$$

$$= -\frac{1}{4u^2} + C = -\frac{1}{4(1+x^2)^2} + C.$$

$$\int x\sqrt{x+1}\,dx$$

double sub

$u = x+1 \rightarrow x = u-1$

$du = dx$

$$= \int u^{1/2}(u-1)\,du$$

$$= \int \left(u^{3/2} - u^{1/2}\right) du$$

$$= \frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2} + C$$

$$= \frac{2}{3}\sqrt{(x+1)^5} - \frac{2}{3}\sqrt{(x+1)^3} + C.$$

$$\int (x+3)\sqrt{x+1}\,dx$$

$u = x+1 \longrightarrow x = u-1$

$du = dx \qquad x+3 = u+2$

$$= \int u^{1/2}(u+2)\,du$$

$$= \int \left(u^{3/2} + 2u^{1/2}\right) du$$

$$= \frac{2}{5}u^{5/2} + \frac{4}{3}u^{3/2} + C$$

$$= \frac{2}{5}\sqrt{(x+1)^5} + \frac{4}{3}\sqrt{(x+1)^3} + C.$$

$$\int x^2 \sqrt{x+1}\, dx$$

$u = x+1 \rightarrow x = u-1$

$du = dx \qquad x^2 = u^2 - 2u + 1$

$$= \int u^{1/2}(u^2 - 2u + 1)\, du$$

etc.

$$\int e^{ax+b}\,dx = \frac{1}{a}\int e^{u}\,du$$

$u = ax+b$ $\qquad = \frac{1}{a}e^{u} + C$

$du = a\,dx$

$\frac{1}{a}du = dx$ $\qquad = \frac{1}{a}e^{ax+b} + C.$

$$\int e^{8x}\,dx = \frac{1}{8}e^{8x} + C.$$

$$\int \sin(4x+3)\,dx = -\frac{1}{4}\cos(4x+3) + C.$$

$$\int \frac{1}{5x-3}\,dx = \frac{1}{5}\ln|5x-3| + C.$$