

## Antidifferentiation

↔ "Integration"

Antidiff.  
Rec. mot.  
Substi.  
Diffy Q.

Summations  $\sum_{i=1}^n$

Approx area

$\lim_{n \rightarrow \infty} \sum_{i=1}^n f(i)$  exact  
area

Definite Integral  
FTC  
↓  
Apps

$$f'(x) = 8x - 3$$

$$f(x) = 4x^2 - 3x + C$$

$$\int (8x - 3) dx = 4x^2 - 3x + C.$$

$$\begin{aligned}& \int \sqrt{x} (2x^3 - 7x + 1) dx \\&= \int x^{1/2} (2x^3 - 7x + 1) dx \\&= \int (2x^{5/2} - 7x^{3/2} + x^{1/2}) dx \\&= 2 \cdot \frac{2}{7} x^{7/2} - 7 \cdot \frac{2}{5} x^{5/2} + \frac{2}{3} x^{3/2} + C \\&= \frac{4}{7} \sqrt{x^7} - \frac{14}{5} \sqrt{x^5} + \frac{2}{3} \sqrt{x^3} + C\end{aligned}$$

$$\int x^{-1} dx = \frac{x^0}{0} + \dots \text{ nope.}$$

$$\int \frac{1}{x} dx = \ln|x| + C$$

### Power Rule for Antiderivatives

$$\int u^n du = \begin{cases} \frac{u^{n+1}}{n+1} + C & n \neq -1 \\ \ln|u| + C & n = -1 \end{cases}$$

$$\begin{aligned}& \int \frac{x^2 + 5x}{x^3} dx \\&= \int \left( \frac{1}{x} + 5x^{-2} \right) dx \\&= \ln|x| + (5) \frac{x^{-1}}{-1} + C \\&= \ln|x| - \frac{5}{x} + C.\end{aligned}$$

" +C"  $\rightarrow$  general sol.

Given  $f'(x) = 12x^2 - 24x + 1$  and  $f(1) = -2$ , find  $f(x)$ .

$$f(x) = 4x^3 - 12x^2 + x + C \quad \text{Initial cond.}$$

$$-2 = 4 - 12 + 1 + C$$

$$5 = C$$

particular  
soln.

$$\therefore f(x) = 4x^3 - 12x^2 + x + 5 \leftarrow$$

$f''(x) = 3x - 7$  find  $f(x)$  and  $f'(1) = 2$  and  $f(1) = 3$

$$f'(x) = \frac{3}{2}x^2 - 7x + C$$

$$f(x) = \frac{1}{2}x^3 - \frac{7}{2}x^2 + Cx + D$$

At any pt  $(x,y)$  on a curve the slope of a tang to the curve is given by  $4x-5$ . If curve contains  $(3,7)$   
find eq. of the curve.

$$f'(x) = 4x - 5$$

$$f(x) = 2x^2 - 5x + C \quad - \text{gen. soln}$$

$$7 = 18 - 15 + C$$

$$4 = C$$

$$\therefore f(x) = 2x^2 - 5x + 4 \quad - \text{part. soln.}$$

$$\int \cos u \, du = \sin u + C$$

$$\int \sin u \, du = -\cos u + C$$

$$\int \sec u \tan u \, du = \sec u + C$$

$$\int \csc u \cot u \, du = -\csc u + C$$

$$\int \sec^2 u \, du = \tan u + C$$

$$\int \csc^2 u \, du = -\cot u + C$$

