Differential Equations--Day 2

$\frac{dy}{dx} = e^y(3x^2 - 6x)$ $(1,0)$

$e^{-y}\,dy = (3x^2 - 6x)\,dx$

$-e^{-y} = x^3 - 3x^2 + C$

$-e^0 = 1 - 3 + C$

$-1 = -2 + C$

$C = 1$

$-e^{-y} = x^3 - 3x^2 + 1$

$e^{-y} = -x^3 + 3x^2 - 1$

$-y = \ln(-x^3 + 3x^2 - 1)$

$y = -\ln(-x^3 + 3x^2 - 1)$

$\frac{dy}{dx} = y\cos x$ $\quad y = 3$ when $x = 0$ find $y = f(x)$

$\frac{1}{y}\,dy = \cos x\,dx$

$\ln|y| = \sin x + C$

$y = e^{\sin x + C}$

$y = Ae^{\sin x}$

$3 = Ae^{\sin 0} \rightarrow A = 3$

$y = 3e^{\sin x}$

$$\frac{dy}{dx} = \frac{1+x}{xy} \qquad y(1)=4 \quad \text{find } y=f(x).$$

$$y\,dy = \frac{1+x}{x}\,dx$$

$$\frac{1}{2}y^2 = \ln|x| + x + C$$

$$8 = \cancel{\ln 1} + 1 + C$$

$$C = 7$$

$$\frac{1}{2}y^2 = \ln|x| + x + 7$$

Aside

$$\int \frac{1+x}{x}\,dx$$

$$= \int \left(\frac{1}{x}+1\right)dx$$

$$y^2 = 2\ln|x| + 2x + 14$$

$$y = \pm\sqrt{2\ln|x| + 2x + 14}$$

Since $y(1) = 4$,

$$y = \sqrt{2\ln|x| + x + 14}$$

$$\frac{dB}{dt} = \frac{1}{5}(100 - B) \qquad B(0) = 20 \text{ find } B(t).$$

$$\frac{1}{100 - B}\, dB = \frac{1}{5}\, dt$$

$$-\ln|100 - B| = \frac{1}{5}t + C$$

$$\ln|100 - B| = -\frac{1}{5}t + D$$

$$100 - B = e^{-\frac{1}{5}t + D}$$

$$100 - B = Ae^{-\frac{1}{5}t}$$

$$B = 100 - Ae^{-\frac{1}{5}t}$$

$$20 = 100 - Ae^{0}$$

$$A = 80$$

$$B(t) = 100 - 80e^{-\frac{1}{5}t}$$