

Differential Equations

$$\frac{dy}{dx} = \frac{2x^2}{3y^3}$$

"find $y = f(x)$ "

$$\int 3y^3 dy = \int 2x^2 dx$$

$$\frac{3}{4}y^4 = \frac{2}{3}x^3 + C$$

$$9y^4 = 8x^3 + D$$

$$\frac{d^2 y}{dx^2} = 4x + 3$$

$$\int dx = x$$

$$\int dw = w$$

$$\frac{dy'}{dx} = 4x + 3$$

$$\int dy' = \int (4x + 3) dx$$

$$\frac{dy}{dx} = 2x^2 + 3x + C$$

$$\int dy = \int (2x^2 + 3x + C) dx$$

$$y = \frac{2}{3}x^3 + \frac{3}{2}x^2 + Cx + D$$

$$\frac{dy}{dx} = \frac{\ln x}{xy + xy^3}$$

$$\frac{dy}{dx} = \frac{\ln x}{x(y + y^3)}$$

$$\int (y + y^3) dy = \int \frac{\ln x}{x} dx$$

$$\frac{1}{2}y^2 + \frac{1}{4}y^4 = \frac{1}{2}\ln^2 x + C$$

$$2y^2 + y^4 = 2\ln^2 x + D.$$

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Aside ☆

$$\int \frac{\ln x}{x} dx$$

$$u = \ln x$$

$$du = \frac{1}{x} dx$$

$$\int u du = \frac{1}{2}u^2 + C$$

$$= \frac{1}{2}(\ln x)^2 + C$$

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$$\frac{dy}{dx} = \frac{3x^2}{e^{2y}}$$

$$f(0) = \frac{1}{2} \quad \text{find } y = f(x).$$

$$\int e^{2y} dy = \int 3x^2 dx$$

$$\frac{1}{2} e^{2y} = x^3 + C$$

$$\frac{1}{2} e = C$$

$$\frac{1}{2} e^{2y} = x^3 + \frac{1}{2} e$$

$$e^{2y} = 2x^3 + e$$

$$2y = \ln(2x^3 + e)$$

$$y = \frac{1}{2} \ln(2x^3 + e)$$

$$y = \ln \sqrt{2x^3 + e}.$$

$$\frac{dy}{dx} = 4xy \quad f(0) = 4 \quad \text{Find } y = f(x).$$

$$\int \frac{1}{y} dy = \int 4x dx$$

$$\ln|y| = 2x^2 + C$$

$$y = e^{2x^2 + C}$$

$$y = Ae^{2x^2}$$

$$4 = Ae^0 \rightarrow A = 4$$

$$\therefore y = 4e^{2x^2}$$

$$\frac{dy}{dx} = \frac{x}{y}$$

$$y(3) = 4$$

find $y = f(x)$.

$$\int y \, dy = \int x \, dx$$

$$\frac{1}{2}y^2 = \frac{1}{2}x^2 + C$$

$$y = \frac{x}{2} + C$$

$$C = \frac{7}{2}$$

$$\frac{1}{2}y^2 = \frac{1}{2}x^2 + \frac{7}{2}$$

$$y^2 = x^2 + 7$$

$$y = \pm \sqrt{x^2 + 7}$$

$$\text{Since } y(3) = 4$$

$$y = \sqrt{x^2 + 7}$$