More on Antidifferentiation and Substitution

$$\int (x^2 - 10x + 25)^8 \, dx$$

$$= \int \left[(x-5)^2 \right]^8 dx$$

$$= \int (x-5)^{16} \, dx$$

$$= \frac{1}{17}(x-5)^{17} + C$$

Rational functions $\longrightarrow$ $°N \geq °D$ by one.

$$\int \frac{x^2+5x-3}{x+4}\,dx$$

$$\begin{array}{r|rrr} -4 & 1 & 5 & -3 \\ & & -4 & -4 \\ \hline & 1 & 1 & -7 \end{array}$$

$$x+1-\frac{7}{x+4}$$

$$= \int \left(x+1-\frac{7}{x+4} \right) dx$$

$$= \frac{1}{2}x^2 + x - 7\ln|x+4| + C$$

$$\log x^2 = 2\log x$$

$$\int \tan x\,dx = \int \frac{\sin x}{\cos x}\,dx$$

$\cancel{u = \sin x}$ $\qquad$ $u = \cos x$

$\cancel{du = \cos x\,dx}$ $\qquad$ $du = -\sin x\,dx$

$$-du = \sin x\,dx$$

$$= -\int \frac{1}{u}\,du$$

$$= -\ln|u| + C = \ln|u^{-1}| + C$$

$$= \ln|\sec x| + C$$

$$\int \tan u\,du = \ln|\sec u| + C.$$

$$\int \cot x \, dx = \int \frac{\cos x}{\sin x} \, dx = \int \frac{1}{u} \, du$$

$u = \sin x$

$du = \cos x \, dx$

$$= \ln|u| + C$$

$$= \ln|\sin x| + C$$

$$\int \cot u \, du = \ln|\sin u| + C$$

$$\int \sec x \, dx = \int \frac{\sec x(\sec x + \tan x)}{\sec x + \tan x} \, dx$$

$$u = \sec x + \tan x$$

$$du = (\sec x \tan x + \sec^2 x) \, dx$$

$$= \int \frac{1}{u} \, du = \ln|u| + C$$

$$= \ln|\sec x + \tan x| + C.$$

$$\int \sec u \, du = \ln|\sec u + \tan u| + C$$

$$\int \csc u \, du = \ln|\csc u - \cot u| + C$$

$$\int x \sec x^2 \, dx$$

$u = x^2$

$du = 2x \, dx$

$\frac{1}{2} du = x \, dx$

$$\frac{1}{2} \int \sec u \, du$$

$= \frac{1}{2} \ln |\sec u + \tan u| + C.$

$= \frac{1}{2} \ln |\sec x^2 + \tan x^2| + C.$

$$\int \cot(3x-5)\,dx = \frac{1}{3}\ln\left|\sin(3x-5)\right| + C$$