Antidifferentiation via Substitution

$$\int u^n\,du = \begin{cases} \dfrac{u^{n+1}}{n+1} + C & n \neq -1 \\ \ln|u| + C & n = -1 \end{cases}$$

$$\int \frac{1}{u}\,du = \ln|u| + C$$

$$\int e^u\,du = e^u + C$$

$$\int \sin u\,du = -\cos u + C$$

$$\int a^u\,du = \frac{a^u}{\ln a} + C$$

$$\int \cos u\,du = \sin u + C$$

$$\int (5x-2)^8\,dx = \frac{1}{5}\int u^8\,du$$

$$u = 5x-2 \qquad = \frac{1}{5}\frac{1}{9}u^9 + C$$

$$du = 5\,dx$$

$$\frac{1}{5}du = dx \qquad = \frac{1}{45}(5x-2)^9 + C.$$

$$\int \cos 5x \, dx = \frac{1}{5}\int \cos u \, du$$

$$u = 5x \qquad = \frac{1}{5}\sin u + C$$

$$du = 5\,dx$$

$$\frac{1}{5}\,du = dx \qquad = \frac{1}{5}\sin 5x + C.$$

$$\int \overset{10}{5}\cos 5x \, dx = \int \cos u \, du = \sin u + C$$

$$= \sin 5x + C$$

$$u = 5x$$

$$du = 5\,dx$$

$$2\,du = 10\,dx$$

$$\int \sin x \sqrt{1-\cos x}\, dx$$

$u = 1 - \cos x$

$du = \sin x \, dx$

$$= \int u^{1/2} \, du$$

$$= \frac{2}{3} u^{3/2} + C$$

$$= \frac{2}{3} \sqrt{(1-\cos x)^3} + C.$$

$$\int \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx = 2\int \sin u\,du$$

$$= -2\cos u + C$$

$u = \sqrt{x}$

$du = \frac{1}{2\sqrt{x}}\,dx$

$2\,du = \frac{1}{\sqrt{x}}\,dx$

$$= -2\cos\sqrt{x} + C.$$

$$\int \frac{x\,dx}{1+x^2} = \frac{1}{2}\int \frac{1}{u}\,du$$

$$u = 1+x^2 \qquad = \frac{1}{2}\ln|u| + C$$

$$du = 2x\,dx$$

$$\frac{1}{2}du = x\,dx \qquad = \frac{1}{2}\ln\left|x^2+1\right| + C.$$

$$\int 3e^x 5^{e^x}\,dx = 3\int 5^u\,du$$

$u = e^x$

$du = e^x\,dx$

$$= 3\cdot\frac{5^u}{\ln 5} + C$$

$$= \frac{3\left(5^{e^x}\right)}{\ln 5} + C.$$

$$\int x\sqrt{x+1}\,dx$$

double sub

$u = x+1 \rightarrow x = u-1$

$du = dx$

$$= \int u^{1/2}(u-1)\,du$$

$$= \int \left(u^{3/2} - u^{1/2}\right)du$$

$$= \frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2} + C$$

$$= \frac{2}{5}\sqrt{(x+1)^5} - \frac{2}{3}\sqrt{(x+1)^3} + C.$$

$$\int (x+3)\sqrt{x+1}\,dx$$

$u = x+1 \longrightarrow x = u-1$

$du = dx \qquad x+3 = u+2$

$$= \int u^{1/2}(u+2)\,du = \int \left(u^{3/2} + 2u^{1/2}\right) du$$

$$= \frac{2}{5}u^{5/2} + 2\cdot\frac{2}{3}u^{3/2} + C$$

$$= \frac{2}{5}\sqrt{(x+1)^5} + \frac{4}{3}\sqrt{(x+1)^3} + C.$$

$$\int x^2 \sqrt{x+3}\, dx$$

$u = x+3 \rightarrow x = u-3$

$du = dx \qquad x^2 = u^2 - 6u + 9$

$$= \int u^{1/2}(u^2 - 6u + 9)\, du$$

$$= \int \left(u^{5/2} - 6u^{3/2} + 9u^{1/2}\right) du$$

$$= \frac{2}{7}u^{7/2} - 6\cdot\frac{2}{5}u^{5/2} + 9\cdot\frac{2}{3}u^{3/2} + C$$

$$= \frac{2}{7}\sqrt{(x+3)^7} - \frac{12}{5}\sqrt{(x+3)^5} + 6\sqrt{(x+3)^3} + C.$$

$$\int e^{5x+2}\,dx$$

$$= \frac{1}{5}e^{5x+2} + C$$

$$\int \frac{1}{4x-3}\,dx$$

$$= \frac{1}{4}\ln|4x-3| + C.$$

$$\int e^{ax+b}\,dx$$

$$u = ax+b$$

$$du = a\,dx$$

$$\frac{1}{a}\,du = dx$$

$$\frac{1}{a}\int e^{u}\,du = \frac{1}{a}e^{u} + C$$

$$= \frac{1}{a}e^{ax+b} + C$$

$$\int (8x-3)^5\,dx = \frac{1}{8}\frac{1}{6}(8x-3)^6 + C$$

3 more known anti-der.

$$\int \frac{1}{\sqrt{a^2-u^2}}\,du = \sin^{-1}\frac{u}{a} + C$$

$$\int \frac{1}{a^2+u^2}\,du = \frac{1}{a}\tan^{-1}\frac{u}{a} + C$$

$$\int \frac{1}{u\sqrt{u^2-a^2}}\,du = \frac{1}{a}\sec^{-1}\frac{u}{a} + C$$