# Differential Equations Day 2

You *MAY NOT* use a calculator.

Consider the differential equation $\frac{dy}{dx} = e^y\left(3x^2 - 6x\right)$. Let $y = f(x)$ be the particular solution to the differential equation that passes through $(1, 0)$.

(a) Write an equation for the line tangent to the graph of $f$ at the point $(1, 0)$. Use the tangent line to approximate $f(1.2)$.

1 $\left.\frac{dy}{dx}\right|_{(1,0)} = e^0(3-6) = -3$

1 $y - 0 = -3(x-1) \rightarrow y = -3(x-1)$

1 $f(1.2) \approx y(1.2) = -3(1.2-1) = -.6$

$$\frac{dy}{dx} = e^y\left(3x^2 - 6x\right)$$

(b) Find $y = f(x)$, the particular solution to the differential equation that passes through $(1, 0)$.

$$\frac{dy}{dx} = e^y(3x^2 - 6x)$$

1 $e^{-y}\,dy = (3x^2 - 6x)\,dx$

1 1 1

$-e^{-y} = x^3 - 3x^2 + C$

1 $-e^0 = 1 - 3 + C$

$-1 = -2 + C$

$1 = C$

$-e^{-y} = x^3 - 3x^2 + 1$

$e^{-y} = -x^3 + 3x^2 - 1$

$-y = \ln(-x^3 + 3x^2 - 1)$

1 $y = -\ln(-x^3 + 3x^2 - 1)$

$$\frac{dB}{dt} = \frac{1}{5}(100 - B)$$

$B(0) = 20$ find $B(t)$

$$\frac{1}{100 - B}\, dB = \frac{1}{5}\, dt$$

$$-\ln|100 - B| = \frac{1}{5}t + C$$

$$\ln|100 - B| = -\frac{1}{5}t + D$$

$$100 - B = e^{-\frac{1}{5}t + D}$$

$$100 - B = Ae^{-\frac{1}{5}t}$$

$$B = 100 - Ae^{-\frac{1}{5}t}$$

$$20 = 100 - Ae^{0}$$

$$A = 80$$

$$B(t) = 100 - 80e^{-\frac{1}{5}t}$$

$$\frac{dB}{dt} = \frac{1}{5}(100 - B), \text{ find } \frac{d^2B}{dt^2}.$$

$$\frac{d^2B}{dt^2} = \frac{1}{5}\left[-\frac{dB}{dt}\right]$$

$$= \frac{1}{5}\left[-\frac{1}{5}(100 - B)\right]$$

$$\frac{dy}{dx} = y\cos x \quad y = 3 \text{ when } x = 0 \quad \text{find } y = f(x).$$

$$\frac{1}{y}\,dy = \cos x \; dx$$

$$\ln|y| = \sin x + C$$

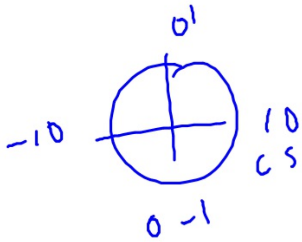
$$y = e^{\sin x + C}$$

$$y = Ae^{\sin x}$$

$$3 = Ae^{\sin 0}$$

$$A = 3$$

$$y = 3e^{\sin x}$$



$$\frac{dy}{dx} = \frac{1+x}{xy} \qquad y(1)=4 \quad \text{find } y=f(x).$$

$$y\,dy = \frac{1+x}{x}\,dx$$

$$\frac{1}{2}y^2 = \ln|x| + x + C$$

$$8 = \cancel{\ln 1} + 1 + C$$

$$7 = C$$

$$\frac{1}{2}y^2 = \ln|x| + x + 7$$

$$y^2 = 2\ln|x| + 2x + 14$$

$$y = \pm\sqrt{2\ln|x| + 2x + 14}$$

<u>Aside</u>

$$\int\left(\frac{1}{x}+1\right)dx$$

$$= \ln|x| + x + C.$$

Since $y(1) = -4$

$$y = -\sqrt{2\ln|x| + 2x + 14}.$$