Differential Equations

"find $y = f(x)$"

$$\frac{dy}{dx} = \frac{2x^2}{3y^3}$$

$$\int 3y^3\,dy = \int 2x^2\,dx$$

$$\frac{3}{4}y^4 = \frac{2}{3}x^3 + C$$

$$9y^4 = 8x^3 + D$$

$$\frac{d^2y}{dx^2} = 4x + 3$$

$$\frac{dy'}{dx} = 4x + 3$$

$$\int dy' = \int (4x + 3)\,dx$$

$$\frac{dy}{dx} = 2x^2 + 3x + C$$

$$\int dy = \int (2x^2 + 3x + C)\,dx$$

$$y = \frac{2}{3}x^3 + \frac{3}{2}x^2 + Cx + D$$

$$\frac{dy}{dx} = \frac{\ln x}{xy + xy^3}$$

$$\frac{dy}{dx} = \frac{\ln x}{x(y + y^3)}$$

$$\int (y + y^3)\,dy = \int \frac{\ln x}{x}\,dx$$

$$\frac{1}{2}y^2 + \frac{1}{4}y^4 = \frac{1}{2}\ln^2 x + C$$

$$2y^2 + y^4 = 2\ln^2 x + D$$

Aside

$$\int \frac{\ln x}{x}\,dx$$

$u = \ln x$

$du = \frac{1}{x}\,dx$

$\int u\,du$

$\frac{1}{2}u^2 + C$

$\frac{1}{2}(\ln x)^2 + C$

$$\frac{dy}{dx} = \frac{3x^2}{e^{2y}} \qquad f(0) = \frac{1}{2} \quad \text{find} \quad y = f(x).$$

$$\int e^{2y}\,dy = \int 3x^2\,dx$$

$$\frac{1}{2}e^{2y} = x^3 + C$$

$$\frac{1}{2}e = C$$

$$\frac{1}{2}e^{2y} = x^3 + \frac{1}{2}e$$

$$e^{2y} = 2x^3 + e$$

$$2y = \ln(2x^3 + e)$$

$$y = \frac{1}{2}\ln(2x^3 + e)$$

$$y = \ln\sqrt{2x^3 + e}$$

$\frac{dy}{dx} = 4xy \qquad f(0) = 4 \quad \text{find} \quad y = f(x).$

$\int \frac{1}{y}\,dy = \int 4x\,dx$

$\ln|y| = 2x^2 + C$

$y = e^{2x^2+C}$

$y = e^{2x^2} \cdot e^{C}$

$y = Ae^{2x^2}$

$4 = Ae^{0}$

$4 = A$

$\therefore \; y = 4e^{2x^2}$

$$\frac{dy}{dx} = \frac{x}{y} \qquad y(3) = 4 \quad \text{find} \quad \frac{dy}{dx}.$$

$$\int y\,dy = \int x\,dx$$

$$\frac{1}{2}y^2 = \frac{1}{2}x^2 + C$$

$$8 = \frac{9}{2} + C$$

$$\frac{7}{2} = C$$

$$\frac{1}{2}y^2 = \frac{1}{2}x^2 + \frac{7}{2}$$

$$y^2 = x^2 + 7$$

$$y = \pm\sqrt{x^2+7}$$

Since $f(3) = 4$

$$y = \sqrt{x^2+7}$$