# Antidifferentiation via Substitution

$$\int u^n \, du = \begin{cases} \dfrac{u^{n+1}}{n+1} + C & n \neq -1 \\ \ln|u| + C & n = -1 \end{cases}$$

$$\int e^u \, du = e^u + C \qquad \int \sin u \, du = -\cos u + C$$

$$\int a^u \, du = \frac{a^u}{\ln a} + C \qquad \int \cos u \, du = \sin u + C.$$

$$\int \frac{1}{\sqrt{a^2-u^2}}\,du = \sin^{-1}\frac{u}{a} + C$$

$$\int \frac{1}{a^2+u^2}\,du = \frac{1}{a}\tan^{-1}\frac{u}{a} + C$$

$$\int \frac{1}{u\sqrt{u^2-a^2}}\,du = \frac{1}{a}\sec^{-1}\frac{u}{a} + C.$$

$$\int (7x+2)^3\,dx = \frac{1}{7}\int u^3\,du$$

$u = 7x+2$

$du = 7\,dx$

$\frac{1}{7}du = dx$

$$= \frac{1}{7}\,\frac{1}{4}u^4 + C$$

$$= \frac{1}{28}(7x+2)^4 + C.$$

$$\int \underline{3}\cos 3x\,\underline{dx} = \int \cos u\,du$$

$u = 3x$

$du = 3\,dx$

$$= \sin u + C$$

$$= \sin 3x + C.$$

$$\int 9\cos 3x\,dx = 3\int \cos u\,du$$

$u = 3x$

$du = 3dx$

$3du = 9dx$

$$= 3\sin 3x + C.$$

$u = 3x$

$du = 3dx$

$\frac{1}{3}du = dx$

$$= \frac{9}{3}\int \cos u\,du$$

$$\int x^2(5+2x^3)^7\,dx = \frac{1}{6}\int u^7\,du$$

$$u = 5+2x^3$$

$$du = 6x^2\,dx$$

$$\frac{1}{6}\,du = x^2\,dx$$

$$= \frac{1}{6}\,\frac{1}{8}u^8 + C$$

$$= \frac{1}{48}(5+2x^3)^8 + C.$$

$$\int \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx = 2\int \sin u\,du$$

$$= -2\cos u + C$$

$$= -2\cos\sqrt{x} + C$$

$u = \sqrt{x}$

$du = \frac{1}{2\sqrt{x}}\,dx$

$2\,du = \frac{1}{\sqrt{x}}\,dx$

$$\int 3e^x 5^{e^x}\,dx = 3\int 5^u\,du$$

$u = e^x$

$du = e^x\,dx$

$$= 3\,\frac{5^u}{\ln 5} + C$$

$$= \frac{3\cdot 5^{e^x}}{\ln 5} + C.$$

$$\int x\sqrt{x+1}\,dx = \int u^{1/2}(u-1)\,du$$

$u = x+1 \rightarrow x = u-1$

$du = dx$

"double sub"

$$= \int \left(u^{3/2} - u^{1/2}\right) du$$

$$= \frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2} + C$$

$$= \frac{2}{5}\sqrt{(x+1)^5} - \frac{2}{3}\sqrt{(x+1)^3} + C.$$

$$\int (x+3)\sqrt{x+1}\,dx$$

$u = x+1 \longrightarrow x = u-1$

$du = dx \qquad x+3 = u+2$

$$= \int u^{1/2}(u+2)\,du = \int \left(u^{3/2} + 2u^{1/2}\right)du$$

$$= \frac{2}{5}u^{5/2} + 2\cdot\frac{2}{3}u^{3/2} + C$$

$$= \frac{2}{5}\sqrt{(x+1)^5} + \frac{4}{3}\sqrt{(x+1)^3} + C.$$

$$\int x^2 (3x+2)^5 \, dx$$

$u = 3x+2 \rightarrow x = \dfrac{u-2}{3}$

$du = 3\,dx$

$\frac{1}{3} du = dx \qquad x^2 = \dfrac{u^2-4u+4}{9}$

$$\frac{1}{3}\frac{1}{9}\int u^5 (u^2-4u+4)\, du$$

$$= \frac{1}{27}\int (u^7 - 4u^6 + 4u^5)\, du = \frac{1}{27}\left[\frac{1}{8}u^8 - \frac{4}{7}u^7 + \frac{4}{6}u^6\right] + C.$$

$$\int e^{3x}\,dx$$

$$= \frac{1}{3}e^{3x} + C$$

$$\int \cos 8x\,dx$$

$$= \frac{1}{8}\sin 8x + C.$$

$$\int e^{ax+b}\,dx$$

$u = ax+b$

$du = a\,dx$

$\frac{1}{a}\,du = dx$

$$\frac{1}{a}\int e^{u}\,du = \frac{1}{a}e^{u} + C$$

$$= \frac{1}{a}e^{ax+b} + C$$

$$\int \frac{1}{5x-2}\,dx = \frac{1}{5}\ln|5x-2| + C.$$

$$\int 3^{5x-7}\,dx = \frac{3^{5x-7}}{5\ln 3} + C.$$