Antidifferentiation ⟷ Integration

Antideriv.

Rect. mot.

Subst.

Summations Σ

Approx areas

$\lim \Sigma$ – ext. area

Def. Int

FTC

Apps of Def Int.

$$f'(x) = x^2$$

$$f(x) = \frac{1}{3}x^3 + C$$

$$\frac{dy}{dx} = x^2$$

$$\int 1\,dy = \int x^2\,dx$$

$$y = \frac{1}{3}x^3 + C$$

$$\int x^2\,dx = \frac{1}{3}x^3 + C$$

$$\int \sqrt{x}\,(x+3)\,dx = \int x^{1/2}(x+3)\,dx$$

$$= \int \left[x^{3/2} + 3x^{1/2} \right] dx$$

$$= \frac{2}{5}x^{5/2} + 3\cdot\frac{2}{3}x^{3/2} + C$$

$$= \frac{2}{5}\sqrt{x^5} + 2\sqrt{x^3} + C.$$

$$\int x^{-1}\,dx = \frac{x^0}{0}$$ nope.

$$\int \frac{1}{x}\,dx = \ln|x| + C$$

Power Rule for Antiderivatives.

$$\int u^n\,du = \begin{cases} \dfrac{u^{n+1}}{n+1} & n \neq -1 \\ \ln|u| & n = -1 \end{cases}$$

$$\int \frac{x^2+3x}{\sqrt{x}}\,dx = \int x^{-1/2}\left(x^2+3x\right)dx$$

$$= \int \left[x^{3/2}+3x^{1/2}\right]dx$$

$$= \frac{2}{5}x^{5/2} + 3\cdot\frac{2}{3}x^{3/2} + C$$

$$= \frac{2}{5}\sqrt{x^5} + 2\sqrt{x^3} + C.$$

$$\int e^x \, dx = e^x + C$$

$$\int \sin x \, dx = -\cos x + C$$

$$\int \cos x \, dx = \sin x + C$$

$$\int \sec x \tan x \, dx = \sec x + C$$

$$\int \sec^2 x \, dx = \tan x + C$$

$$\int \csc x \cot x \, dx = -\csc x + C$$

$$\int \csc^2 x \, dx = -\cot x + C$$

"+C" general soln.

Given $f'(x) = 12x^2 - 24x + 1$ and $f(1) = -2$, find $f(x)$.

$$f(x) = 4x^3 - 12x^2 + x + C$$
$$-2 = 4(1)^3 - 12(1)^2 + 1 + C$$
$$-2 = 4 - 12 + 1 + C$$
$$5 = C$$

$$\therefore f(x) = 4x^3 - 12x^2 + x + 5$$

particular soln.

$f''(x) = 3x + 2$ find $f(x)$. [if $f'(1) = 3$ & $f(1) = 2$]

$f'(x) = \frac{3}{2}x^2 + 2x + C$ ✓

$f(x) = \frac{3}{2}\frac{1}{3}x^3 + 2 \cdot \frac{1}{2}x^2 + Cx + D$

$f(x) = \frac{1}{2}x^3 + x^2 + Cx + D.$

At any pt (x,y) on a curve the slope of a tangent is given by $4x-5$. If curve pass thru $(3,7)$, find an eq. for the curve.

$f'(x) = 4x - 5$

$f(x) = 2x^2 - 5x + C$

$7 = 2(3)^2 - 15 + C$

$7 = 18 - 15 + C$

$4 = C$

$\therefore f(x) = 2x^2 - 5x + 4$

$\frac{dy}{dx} = 4x - 5$

$\int dy = \int (4x-5)\,dx$

$y = 4x - 5 + C$