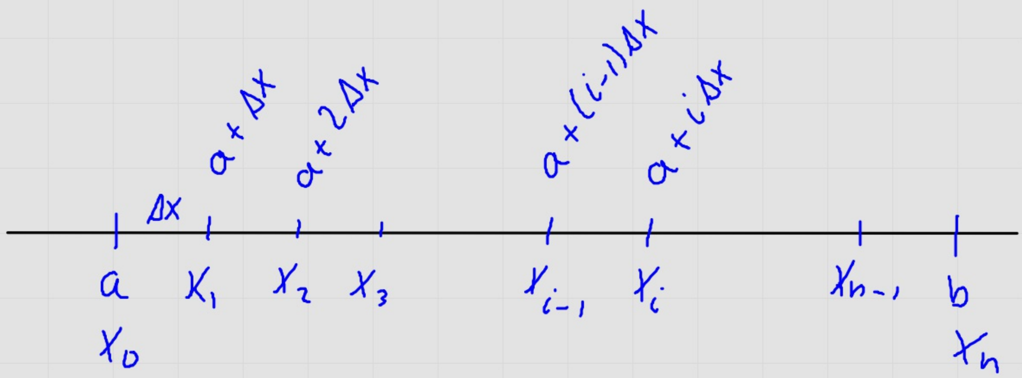
Exact Area via the Limit of a Riemann Sum



$$A_R \approx \sum_{i=1}^{n} f(x_i)\Delta x$$

$$A_L \approx \sum_{i=0}^{n-1} f(x_i)\Delta x$$

$$\approx \sum_{i=1}^{n} f(x_{i-1})\Delta x$$

$$A \approx \sum_{i=1}^{n} f(c_i)\Delta x$$

$c_i = x_i$ R.S.

$C_i = x_{i-1}$ L.S.

$$A = \lim_{n \to \infty} \sum_{i=1}^{n} f(c_i)\Delta x$$

$c_i = x_i = a + i\Delta x$ R.S.

$c_i = x_{i-1} = a + (i-1)\Delta x$ L.S.

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} f(\overset{x_i}{a+i\Delta x})\,\Delta x$$

Bounds: $f(x) = 5x+1$, $x=2$, $X=6$, x-axis.

$$\Delta x = \frac{6-2}{n} = \frac{4}{n} \qquad x_i = 2 + i\Delta x$$

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} f(2+i\Delta x)\,\Delta x$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left[5(2+i\Delta x)+1\right]\Delta x$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left[11\Delta x + 5i(\Delta x)^2\right]$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left[11\Delta x + 5i(\Delta x)^2 \right]$$

$\frac{16}{n^2} \cdot 5$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left[\frac{44}{n} + \frac{80}{n^2} i \right]$$

$$= \lim_{n\to\infty} \left[44 + \frac{80}{n^2} \frac{n^2+n}{2} \right]$$

$$= 44 + 40(1)$$

$$= 84$$

Bounds: $f(x) = x^2$, $x = 1$, $x = 3$, x-axis

$$\Delta x = \frac{2}{n} \qquad x_i = 1 + i\Delta x$$

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} f(1 + i\Delta x)\Delta x$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left[1 + 2i\Delta x + i^2(\Delta x)^2\right]\Delta x$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left[\Delta x + 2i(\Delta x)^2 + i^2(\Delta x)^3\right]$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left[\frac{2}{n} + \frac{8}{n^2}i + \frac{8}{n^3}i^2\right]$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left[\frac{2}{n} + \frac{8}{n^2} i + \frac{8}{n^3} i^2 \right]$$

$$= \lim_{n\to\infty} \left[2 + \frac{8}{n^2} \frac{n^2+n}{2} + \frac{8}{n^3} \frac{2n^3+3n^2+n}{6} \right]$$

$$= 2 + 4(1) + \frac{4}{3}(2)$$

$$= 6 + \frac{8}{3}$$

$$= \frac{26}{3}.$$

Bounds: $f(x) = x^2 + x$, $x = 2$, $x = 5$, x-axis

$$\Delta x = \frac{3}{n} \qquad x_i = 2 + i\Delta x$$

$$A = \lim_{n \to \infty} \sum_{i=1}^{n} f(2 + i\Delta x)\Delta x$$

$$= \lim_{n \to \infty} \sum_{i=1}^{n} \left[6\Delta x + 5i(\Delta x)^2 + i^2(\Delta x)^3 \right]$$

$$= \lim_{n \to \infty} \sum_{i=1}^{n} \left[\frac{18}{n} + \frac{45}{n^2} i + \frac{27}{n^3} i^2 \right]$$

$$= \lim_{n \to \infty} \left[18 + \frac{45}{n^2} \frac{n^2 + n}{2} + \frac{27}{n^3} \frac{2n^3 + 3n^2 + n}{6} \right]$$

$$= 18 + \frac{45}{2}(1) + \frac{9}{2}(2)$$

$$= \frac{99}{2}$$