Differential Equations--Day 2

$$\frac{dy}{dx} = e^{y}(3x^2 - 6x)$$

(1) $\left.\frac{dy}{dx}\right|_{(1,0)} = e^0(3-6) = -3$

$y - 0 = -3(x-1)$

(1) $y = -3(x-1)$

(1) $f(1.2) \approx y(1.2) = -3(1.2-1) = -.600$

$$\frac{dy}{dx} = e^{y}(3x^2 - 6x) \qquad (1,0)$$

1 $e^{-y}\,dy = (3x^2 - 6x)\,dx$

2+1 $-e^{-y} = x^3 - 3x^2 + C$

$-e^{0} = 1 - 3 + C$

$-1 = -2 + C$

1 $1 = C$

$-e^{-y} = x^3 - 3x^2 + 1$

$e^{-y} = -x^3 + 3x^2 - 1$

$-y = \ln(-x^3 + 3x^2 - 1)$

1 $y = -\ln(-x^3 + 3x^2 - 1)$

$$\frac{dy}{dx} = y\cos x \quad y = 3 \text{ when } x = 0 \text{ find } y = f(x).$$

$$\frac{1}{y}\,dy = \cos x\,dx$$

$$\ln|y| = \sin x + C$$

$$y = e^{\sin x + C}$$

$$y = Ae^{\sin x}$$

$$3 = A$$

$$y = 3e^{\sin x}$$

$$\frac{dy}{dx} = \frac{1+x}{xy} \qquad y(1)=4 \quad \text{find } y=f(x).$$

$$y\,dy = \frac{1+x}{x}\,dx$$

$$y\,dy = \left(\frac{1}{x}+1\right)dx$$

$$\frac{1}{2}y^2 = \ln|x| + x + C$$

$$8 = \cancel{\ln|1|} + 1 + C$$

$$7 = C$$

$$\frac{1}{2}y^2 = \ln|x| + x + 7$$

$$y^2 = 2\ln|x| + 2x + 14$$

$$y = \pm\sqrt{2\ln|x| + 2x + 14}$$

Since $f(1)=4$,

$$y = \sqrt{2\ln|x| + 2x + 14}.$$

$$\frac{dy}{dx} = 4x^3(2y+1) \quad y(1)=2 \quad \text{find } y = f(x).$$

$$\frac{1}{2y+1}\,dy = 4x^3\,dx$$

$$\frac{1}{2}\ln|2y+1| = x^4 + C$$

$$\ln|2y+1| = 2x^4 + D$$

$$2y+1 = e^{2x^4+D}$$

$$2y+1 = Ae^{2x^4}$$

$$y = \frac{Ae^{2x^4} - 1}{2}$$

$$2 = \frac{Ae^2 - 1}{2}$$

$$4 = Ae^2 - 1$$

$$5 = Ae^2 \rightarrow A = \frac{5}{e^2}$$

$$y = \frac{\frac{5}{e^2}e^{2x^4} - 1}{2}$$

$$y = \frac{5e^{2x^4} - e^2}{2e^2}$$

$$\frac{dB}{dt} = \frac{1}{5}(100 - B)$$

$$\frac{1}{100 - B}\, dB = \frac{1}{5}\, dt$$

$$\frac{1}{-1} \ln|100 - B| = \frac{1}{5} t + C$$

$$\ln|100 - B| = -\frac{1}{5} t + D$$

$$100 - B = Ae^{-\frac{1}{5}t}$$

$$\frac{dy}{dx} = 2y^3$$

$$y^{-3}\, dy = 2\, dx$$

$$\frac{y^{-2}}{-2} = 2x + C$$

$$-\frac{1}{2y^2} = 2x + C$$