Differential Equations

"find $y = f(x)$"

$$\frac{dy}{dx} = \frac{2x^2}{3y^3}$$

$$\int 3y^3 \, dy = \int 2x^2 \, dx$$

$$\frac{3}{4}y^4 = \frac{2}{3}x^3 + C$$

$$9y^4 = 8x^3 + D$$

$$\frac{d^2y}{dx^2} = 4x + 3$$

$$\frac{dy'}{dx} = 4x + 3$$

$$\int dy' = \int (4x + 3)\, dx$$

$$\frac{dy}{dx} = 2x^2 + 3x + C$$

$$\int dy = \int (2x^2 + 3x + C)\, dx$$

$$y = \frac{2}{3}x^3 + \frac{3}{2}x^2 + Cx + D$$

$$\frac{dy}{dx} = \frac{\ln x}{xy + xy^3}$$

$$\frac{dy}{dx} = \frac{\ln x}{x(y+y^3)}$$

$$(y+y^3)\,dy = \frac{\ln x}{x}\,dx$$

$$\frac{1}{2}y^2 + \frac{1}{4}y^4 = \frac{1}{2}\ln^2 x + C$$

$$2y^2 + y^4 = 2\ln^2 x + D$$

☆ Aside ☆

$$\int \frac{(\ln x)^1}{x}\,dx$$

$$u = \ln x$$

$$du = \frac{1}{x}\,dx$$

$$\int u\,du$$

$$= \frac{1}{2}u^2 + C$$

$$= \frac{1}{2}(\ln x)^2 + C$$

$$\frac{dy}{dx} = \frac{3x^2}{e^{2y}} \quad f(0) = \frac{1}{2} \quad \text{find } y = f(x).$$

$$\int e^{2y}\,dy = \int 3x^2\,dx$$

$$\frac{1}{2}e^{2y} = x^3 + C$$

$$\frac{1}{2}e = C$$

$$\frac{1}{2}e^{2y} = x^3 + \frac{1}{2}e$$

$$e^{2y} = 2x^3 + e$$

$$\ln(e^{2y}) = \ln(2x^3 + e)$$

$$2y = \ln(2x^3 + e)$$

$$y = \frac{1}{2}\ln(2x^3 + e)$$

$$y = \ln\sqrt{2x^3 + e}$$

$$\frac{dy}{dx} = 4xy \qquad f(0) = 4, \text{ find } y = f(x).$$

$$\int \frac{1}{y}\, dy = \int 4x \, dx$$

$$\ln|y| = 2x^2 + C$$

$$y = e^{2x^2 + C}$$

$$y = e^{2x^2} e^{C}$$

$$y = Ae^{2x^2}$$

$$4 = Ae^{0}$$

$$4 = A$$

$$\therefore y = 4e^{2x^2}$$

$$\frac{dy}{dx} = \frac{x}{y}$$

$y(3) = 4$ find $y = f(x)$

$$\int y\, dy = \int x\, dx$$

$$\frac{1}{2}y^2 = \frac{1}{2}x^2 + C$$

$$8 = \frac{9}{2} + C$$

$$C = \frac{7}{2}$$

$$\frac{1}{2}y^2 = \frac{1}{2}x^2 + \frac{7}{2}$$

$$y^2 = x^2 + 7$$

$$y = \pm\sqrt{x^2 + 7}$$

Since $y(3) = 4$,

$$y = \sqrt{x^2 + 7}$$