More on Antidifferentiation and Substitution

$$\int (x^2+4x+4)^{\frac{2}{3}}\,dx$$

$$= \int \left[(x+2)^2\right]^{2/3} dx$$

$u = x+2$

$du = dx$

$$\int (u^2)^{2/3}\,du$$

$$= \int u^{4/3}\,du$$

$$= \frac{3}{7}u^{7/3} + C$$

$$= \int (x+2)^{4/3}\,dx$$

$$= \frac{3}{7}(x+2)^{7/3} + C$$

$$= \frac{3}{7}\sqrt[3]{(x+2)^7} + C$$

Rational function $\frac{\text{poly}}{\text{poly}}$

$$f(x) = \frac{x^2+5x-3}{x-2} \qquad \cancel{f(x) = \frac{\sin x}{x+3}}$$

$^\circ N \geq {}^\circ D \longrightarrow$ divide D into N

$$\int \frac{x^2+5x-3}{x-2}\,dx$$

$$\begin{array}{r|rrr} 2 & 1 & 5 & -3 \\ & & 2 & 14 \\ \hline & 1 & 7 & 11 \end{array}$$

$$= \int\left(x+7+\frac{11}{x-2}\right)dx$$

$$= \frac{1}{2}x^2 + 7x + 11\ln|x-2| + C.$$

$$\int \tan x \, dx = \int \frac{\sin x}{\cos x} \, dx$$

$u = \cos x$

$du = -\sin x \, dx$

$-du = \sin x \, dx$

$\int \tan u \, du = \ln|\sec u| + C$

$$= -\int \frac{1}{u} \, du$$

$$= -\ln|u| + C = \ln|u^{-1}| + C$$

$$= \ln|\sec x| + C$$

$$\int \cot x \, dx = \int \frac{\cos x}{\sin x} \, dx$$

$u = \sin x$

$du = \cos x \, dx$

$$= \int \frac{1}{u} \, du$$

$$= \ln|u| + C$$

$$= \ln|\sin x| + C$$

$$\int \cot u \, du = \ln|\sin u| + C$$

$$\int \sec x \, dx = \int \frac{\sec x(\sec x + \tan x)}{\sec x + \tan x} \, dx$$

$$u = \sec x + \tan x$$

$$du = (\sec x \tan x + \sec^2 x) \, dx$$

$$= \int \frac{1}{u} \, du$$

$$= \ln|u| + C$$

$$= \ln|\sec x + \tan x| + C$$

$$\int \sec u \, du = \ln|\sec u + \tan u| + C.$$

$$\int \csc u \, du = \ln|\csc u - \cot u| + C$$

$$\int \underline{e}^x \tan e^x \, \underline{d}x$$

$u = e^x$

$du = e^x \, dx$

$$= \int \tan u \, du$$

$$= \ln|\sec u| + C$$

$$= \ln|\sec e^x| + C.$$

$$\int \sec u \, du = \ln|\sec u + \tan u| + C$$

$$\int \sec(ax+b) = \frac{1}{a}\ln|\sec(ax+b) + \tan(ax+b)| + C$$

$$\int \tan(3x-2)\,dx = \frac{1}{3}\ln|\sec(3x-2)| + C$$