Antidifferentiation via Substitution

$$\int u^n \, du = \begin{cases} \dfrac{u^{n+1}}{n+1} + C & n \neq -1 \\ \ln|u| + C & n = -1 \end{cases}$$

$$\int e^u \, du = e^u + C$$

$$\int a^u \, du = \frac{a^u}{\ln a} + C$$

$$\int \frac{1}{\sqrt{a^2-u^2}}\,du = \sin^{-1}\frac{u}{a} + C$$

$$\int \frac{1}{a^2+u^2}\,du = \frac{1}{a}\tan^{-1}\frac{u}{a} + C$$

$$\int \frac{1}{u\sqrt{u^2-a^2}}\,du = \frac{1}{a}\sec^{-1}\frac{u}{a} + C$$

$$\int \sin u\,du = -\cos u + C$$

$$\int \cos u\,du = \sin u + C$$

$$\int (7x+3)^5 \, dx = \frac{1}{7}\int u^5 \, du$$

$u = 7x+3$

$$= \frac{1}{7}\,\frac{1}{6}\,u^6 + C$$

$$= \frac{1}{42}(7x+3)^6 + C$$

$du = 7\,dx$

$\frac{1}{7}\,du = dx$

$$\int \sin 5x \, dx$$

$$u = 5x$$

$$du = 5 \, dx$$

$$\frac{1}{5} du = dx$$

$$= \frac{1}{5} \int \sin u \, du$$

$$= -\frac{1}{5} \cos u + C$$

$$= -\frac{1}{5} \cos 5x + C$$

$$\int 5 \sin 5x \, dx$$

$$u = 5x$$

$$du = 5 \, dx$$

$$= \int \sin u \, du$$

$$= -\cos u + C$$

$$= -\cos 5x + C.$$

$$\int 10 \cos 2x \, dx$$

$u = 2x$

$du = 2\,dx$

$\frac{1}{2}\,du = dx$

$= 10 \cdot \frac{1}{2} \int \cos u \, du$

$= 5 \sin u + C$

$= 5 \sin 2x + C.$

$u = 2x$

$du = 2\,dx$

$5\,du = 10\,dx$

$= 5 \int \cos u \, du$

$= 5 \sin 2x + C.$

$$\int x \cos x^2 \, dx$$

$u = x^2$

$du = 2x \, dx$

$\frac{1}{2} du = x \, dx$

$= \frac{1}{2} \int \cos u \, du$

$= \frac{1}{2} \sin u + C$

$= \frac{1}{2} \sin x^2 + C.$

$$\int 3e^{5x} \, dx$$

$u = 5x$

$du = 5 \, dx$

$\frac{1}{5} du = dx$

$= \frac{3}{5} \int e^u \, du$

$= \frac{3}{5} e^u + C$

$= \frac{3}{5} e^{5x} + C.$

$$\int \frac{x}{x^2+1}\,dx$$

$$u = x^2+1$$

$$du = 2x\,dx$$

$$\frac{1}{2}\,du = x\,dx$$

$$= \frac{1}{2}\int \frac{1}{u}\,du$$

$$= \frac{1}{2}\ln|u| + C$$

$$= \frac{1}{2}\ln\left|x^2+1\right| + C.$$

$$\int \frac{4x^2\,dx}{(1-8x^3)^4}$$

$$u = 1-8x^3$$

$$du = -24x^2\,dx$$

$$-\frac{1}{6}\,du = 4x^2\,dx$$

$$-\frac{1}{6}\int u^{-4}\,du$$

$$= -\frac{1}{6}\,\frac{u^{-3}}{-3} + C$$

$$= \frac{1}{18(1-8x^3)^3} + C$$

$$\int x\sqrt{x+1}\,dx$$ "double sub"

$u = x+1 \rightarrow x = u-1$

$du = dx$

$$\int u^{1/2}(u-1)\,du$$

$$= \int \left[u^{3/2} - u^{1/2}\right] du = \frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2} + C$$

$$= \frac{2}{5}\sqrt{(x+1)^5} - \frac{2}{3}\sqrt{(x+1)^3} + C.$$

$$\int (x+3)\sqrt{x+1}\,dx$$

$u = x+1 \rightarrow x = u-1$

$du = dx \qquad x+3 = u+2$

$$\int u^{1/2}(u+2)\,du$$

$$\int \left(u^{3/2} + 2u^{1/2}\right)du = \frac{2}{5}u^{5/2} + 2\cdot\frac{2}{3}u^{3/2} + C$$

$$= \frac{2}{5}\sqrt{(x+1)^5} + \frac{4}{3}\sqrt{(x+1)^3} + C.$$

$$\int x^2 \sqrt{x+1}\, dx$$

$u = x + 1 \longrightarrow x = u - 1$

$du = dx \qquad x^2 = u^2 - 2u + 1$

$$\int u^{1/2}\left(u^2 - 2u + 1\right) du$$

$$\int e^{5x+1}\,dx$$

$$u = 5x+1$$

$$du = 5\,dx$$

$$\frac{1}{5}du = dx$$

$$= \frac{1}{5}\int e^{u}\,du$$

$$= \frac{1}{5}e^{5x+1} + C.$$

$$\int \cos(ax+b)\,dx$$

$$u = ax+b$$

$$du = a\,dx$$

$$\frac{1}{a}du = dx$$

$$\frac{1}{a}\int \cos u\,du$$

$$\frac{1}{a}\sin u + C$$

$$\frac{1}{a}\sin(ax+b) + C.$$

$$\int \cos 5x \, dx = -\frac{1}{5} \sin 5x + c$$

$$\int (8x-2)^4 \, dx = \frac{1}{5}\frac{1}{8}(8x-2)^5 + c$$

$$\int \frac{1}{3x+2} \, dx = \frac{1}{3} \ln|3x+2| + c \, .$$