

# Antidifferentiation

Integration

Antiderivatives

Differential Eq.  
Slope fields

Definite Integral  $\leftarrow$

FTC  $\rightarrow$  area  
vol.  
accumulations

Power Rule for Deriv  $\rightarrow$   $\frac{x^{10}}{10x^9}$

$$f'(x) = x^3$$

$$f(x) = \frac{x^4}{4} + C$$

$$\int x^3 dx = \frac{1}{4} x^4 + C$$

$$\int (\sqrt{x} + x^5) dx = \int (x^{1/2} + x^5) dx$$

$$= \frac{2}{3} x^{3/2} + \frac{1}{6} x^6 + C$$

$$= \frac{2}{3} \sqrt{x^3} + \frac{1}{6} x^6 + C.$$

$$\begin{aligned}\int \frac{x^2 + 5x}{\sqrt{x}} dx &= \int x^{-1/2} (x^2 + 5x) dx \\ &= \int [x^{3/2} + 5x^{1/2}] dx \\ &= \frac{2}{5} x^{5/2} + 5 \cdot \frac{2}{3} x^{3/2} + C \\ &= \frac{2}{5} \sqrt{x^5} + \frac{10}{3} \sqrt{x^3} + C.\end{aligned}$$

$$\int x^{-1} dx = \frac{x^0}{0}$$

$$\int \frac{1}{x} dx = \ln|x| + C$$

Power Rule for Antideriv

$$\int u^n du = \begin{cases} \frac{u^{n+1}}{n+1} & n \neq -1 \\ \ln|u| & n = -1 \end{cases}$$

$$\int \left( t - \frac{1}{t} \right) dt = \frac{1}{2} t^2 - \ln|t| + C.$$

$$\int (e^x + 6) dx = e^x + 6x + C.$$

$$\int \sin x dx = -\cos x + C$$

$$\int \cos x dx = \sin x + C$$

$$\int \sec x \tan x dx = \sec x + C$$

$$\int \sec^2 x dx = \tan x + C$$

$$\int \csc^2 x \, dx = -\cot x + C$$

$$\int \csc x \cot x \, dx = -\csc x + C$$

All "general" solns so far.

Given  $f'(x) = 12x^2 - 24x + 1$  and  $f(1) = -2$ , find  $f$ .

$$f(x) = 4x^3 - 12x^2 + x + C$$

$$-2 = 4(1)^3 - 12(1)^2 + 1 + C$$

$$-2 = 4 - 12 + 1 + C$$

$$-2 = -7 + C$$

$$5 = C$$

$$\therefore f(x) = 4x^3 - 12x^2 + x + 5.$$

$$f''(x) = 3x - 2 \quad \text{find } f.$$

$$f'(1) = 2$$

$$f(1) = 3$$

$$f'(x) = \frac{3}{2}x^2 - 2x + C \quad \checkmark$$

$$f(x) = \frac{1}{2}x^3 - x^2 + Cx + D$$

$$f(x) = \frac{1}{2}x^3 - x^2 + Cx + D$$



At any pt  $(x, y)$  on a curve, the slope of a tangent is given by  $4x - 5$ . If curve passes thru  $(3, 7)$ , find the eq. of the curve.

$$f'(x) = 4x - 5$$

$$f(x) = 2x^2 - 5x + C$$

$$7 = 2(3)^2 - 15 + C$$

$$7 = 18 - 15 + C$$

$$4 = C$$

$$\therefore f(x) = 2x^2 - 5x + 4$$