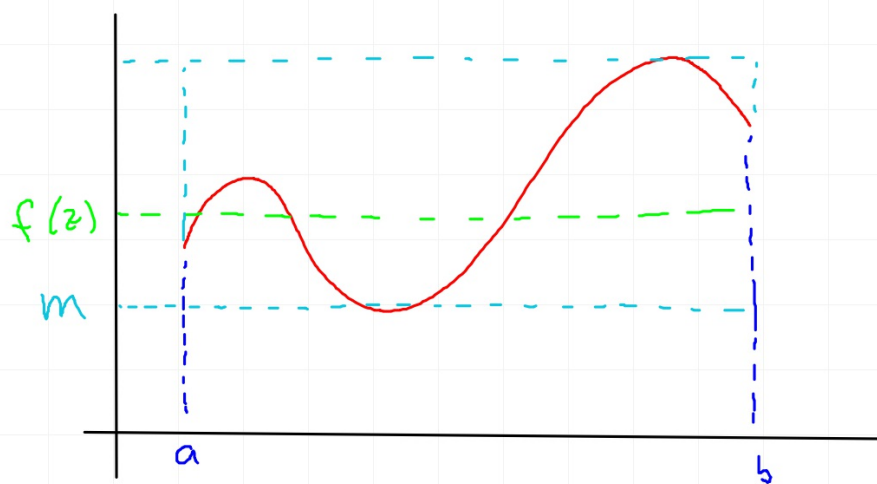


Average Value

Average value of a function on an interval...NOT to be confused with average rate of change in a function on an interval



$$m(b-a) \leq \int_a^b f(x) dx \leq M(b-a)$$

$$\int_a^b f(x) dx \in [m(b-a), M(b-a)]$$

$$f(z)(b-a) = \int_a^b f(x) dx$$

$$f(z) = \frac{1}{b-a} \int_a^b f(x) dx$$

the avg. val.
of f on $[a, b]$.

Avg rate of change in f
on $[a, b]$

$$\frac{f(b) - f(a)}{b - a}$$

Find a closed interval that contains the value of $\int_{\frac{1}{2}}^4 (x^3 - 6x^2 + 9x + 1) dx$ without integrating.

$$f(x) = x^3 - 6x^2 + 9x + 1$$

$$f'(x) = 3x^2 - 12x + 9$$

$$f' \geq 0 \forall x$$

$$f'(x) = 0 \Rightarrow x = 1 \text{ or } x = 3$$

$$f\left(\frac{1}{2}\right) = 4.125$$

$$f(4) = 5 \rightarrow 5\left(4 - \frac{1}{2}\right) = \frac{35}{2}$$

$$f(1) = 5$$

$$f(3) = 1 \rightarrow 1\left(4 - \frac{1}{2}\right) = \frac{7}{2}$$

$$\therefore \int_{\frac{1}{2}}^4 (x^3 - 6x^2 + 9x + 1) dx \in \left[\frac{7}{2}, \frac{35}{2}\right]$$

Find the avg value of $f(x) = 4 - x^2$ on $[0, 2]$.

$$f_{\text{avg}} = \frac{1}{2-0} \int_0^2 (4-x^2) dx$$

$$= \frac{1}{2} \left[4x - \frac{1}{3}x^3 \right]_0^2$$

$$= \frac{1}{2} \left[\left(8 - \frac{8}{3} \right) - (0) \right]$$

$$= \frac{8}{3}$$

$$4 - x^2 = \frac{8}{3}$$

$$V_{avg} = \frac{s(t_2) - s(t_1)}{t_2 - t_1}$$

$$V_{avg} = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} v(t) dt$$