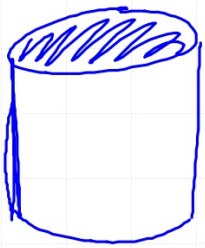
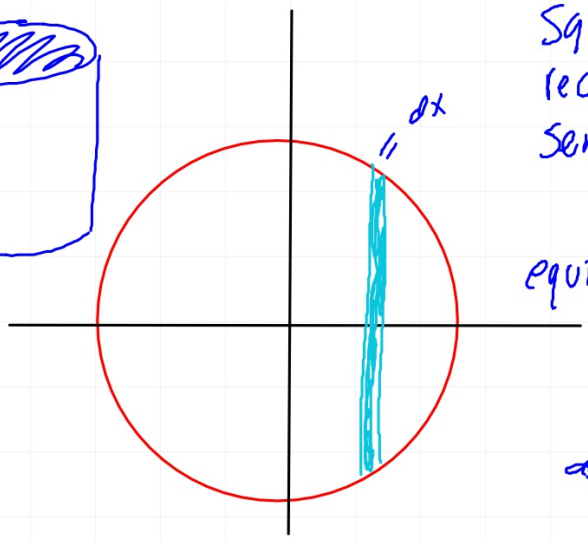
Volumes of Solids with Known Cross Sections

(Slicing)





Squares

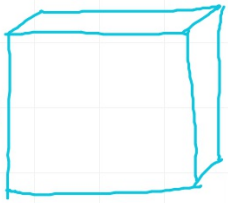
rectangles of given ht $(3-x)$

Semicircles

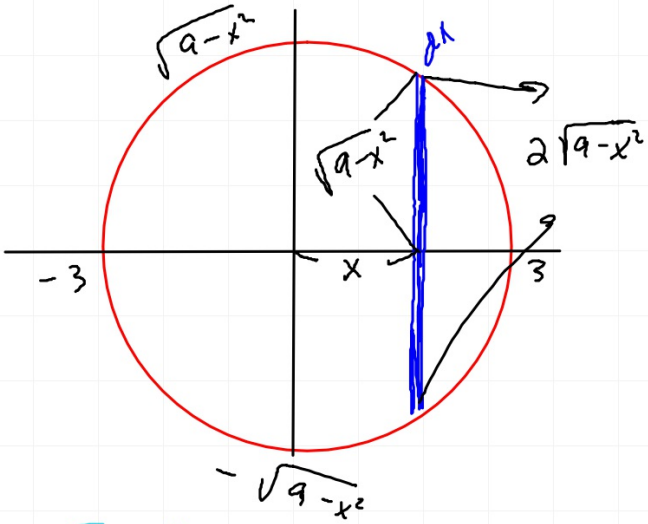
$\frac{\pi}{2} r^2$

equilateral Δ's

$\frac{\sqrt{3}}{4} b^2$



Base: $x^2+y^2=9 \rightarrow y=\pm\sqrt{9-x^2}$



SQUARES

$$V=\int_{-3}^{3}\left[2\sqrt{9-x^2}\right]^2 dx$$

Semicircl

$$V=\frac{\pi}{2}\int_{-3}^{3}\left[\sqrt{9-x^2}\right]^2 dx$$

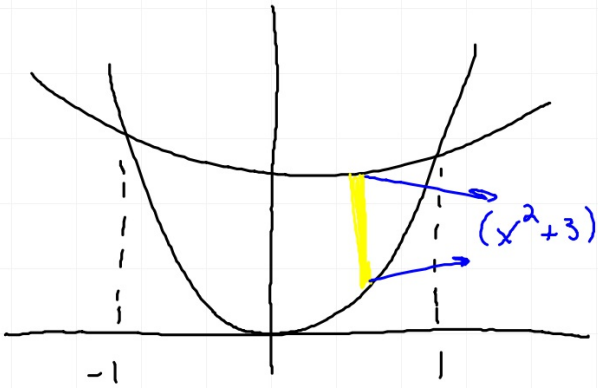
Eq. Δ's

$$V=\frac{\sqrt{3}}{4}\int_{-3}^{3}\left[2\sqrt{9-x^2}\right]^2 dx$$

Red. of Ut 8

$$V=\int_{-3}^{3}(8)\,2\sqrt{9-x^2}\,dx$$

Base: $y = 4x^2$ and $y = x^2 + 3$ CS: squares



Isect

$4x^2 = x^2 + 3 \rightarrow x = -1$ or $x = 1$.

$$V = \int_{-1}^{1} \left[(x^2+3) - 4x^2\right]^2 dx = 9.600$$

CS: semi

$$V = \frac{\pi}{2}\int_{-1}^{1} \left[\frac{(x^2+3) - 4x^2}{2}\right]^2 dx = 3.770$$