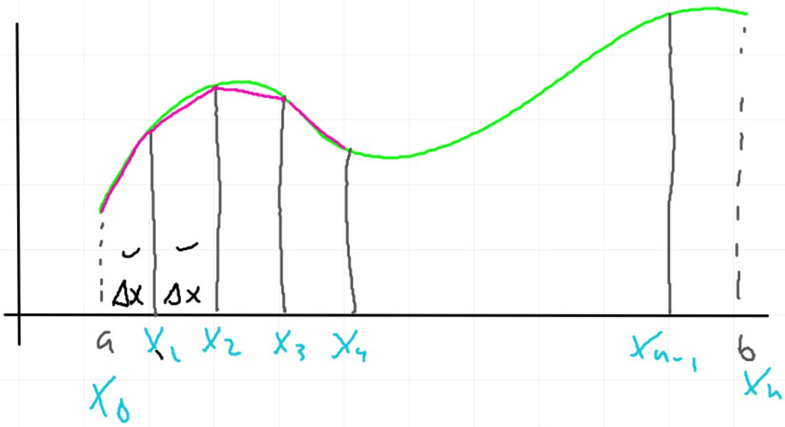
The Trapezoid Rule

(Another way to approximate area/estimate the value of a definite integral.)



$$\int_a^b f(x)\,dx \approx \frac{1}{2}\left[f(x_0)+f(x_1)\right]\Delta x + \frac{1}{2}\left[f(x_1)+f(x_2)\right]\Delta x$$

$$+\frac{1}{2}\left[f(x_2)+f(x_3)\right]\Delta x + \cdots + \frac{1}{2}\left[f(x_{n-1})+f(x_n)\right]\Delta x$$

$$\int_a^b f(x)\,dx \approx \frac{b-a}{2n}\left[f(x_0)+2f(x_1)+2f(x_2)+\dots+2f(x_{n-1})+f(x_n)\right]$$

$$\int_a^b f(x)\,dx = \frac{b-a}{2n}\sum_{i=0}^{n} k_i\, f(x_i)$$

$\hookrightarrow h_i = 1$ a 2

y1 = func.

seq(x, x, a, b, Δx) $\rightarrow$ a

y1(a) $\rightarrow$ b

$\{1, 2, 2, \ldots 2, 1\} \rightarrow k$

b * k $\rightarrow$ c

sum(c)

$* \frac{\Delta x}{2}$

Est $\int_3^7 \sqrt{x-1}\,dx$ using 5 subintervals and TR.

$\Delta x = \frac{4}{5} = .800 \qquad \frac{b-a}{2n} = .400$

	$f(x_i)$	h_i	$h_i f(x_i)$
$x_0 = 3$	1.414	1	1.414
$x_1 = 3.800$	1.637	2	3.347
$x_2 = 4.600$	1.897	2	3.795
$x_3 = 5.400$	2.098	2	4.195
$x_4 = 6.200$	2.280	2	4.561
$x_5 = 7$	2.449	1	2.449

$$\int_3^7 \sqrt{x-1}\,dx \approx (.4)(19.761) \approx 7.904$$

1. $$\int_a^b f(x)\,dx > T$$

2.

$\frac{1}{2}(13)$

$\frac{1}{2}(9)$

$\frac{1}{2}(23)$

$\frac{45}{2}$

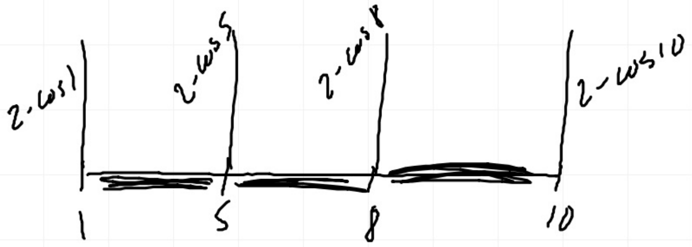
$x^3 - 6x + 9$

$1 - 6 + 9$

$8 - 12 + 9$

$27 - 18 + 9$

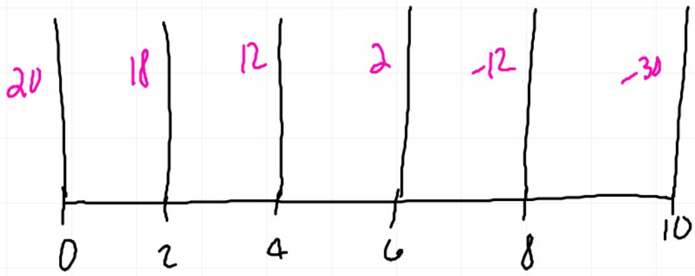
3



$$\frac{1}{2}(4-\cos 1-\cos 5)(4)$$

$$\frac{1}{2}(4-\cos 5-\cos 8)(3)$$

$$\frac{1}{2}(4-\cos 8-\cos 10)(2)$$



$$\cancel{\tfrac{1}{2}}(38)(\cancel{2})$$

$$\cancel{\tfrac{1}{2}}(30)(\cancel{2})$$

$$\cancel{\tfrac{1}{2}}(14)\cancel{2}$$

$$\cancel{\tfrac{1}{2}}(-10)\cancel{2}$$

$$\cancel{\tfrac{1}{2}}(-42)\cancel{2}$$

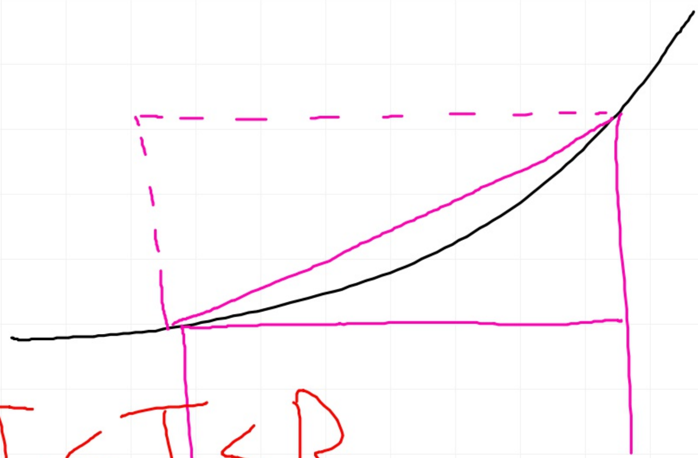
$$I = \int_a^b f(x)\,dx \qquad f'>0 \quad f''>0$$

R RRS

L LRS

T TS

Put I, R, L, T in order from smallest to largest.



$L < I < T < R$

27 ML no cal 55
18 ML cal 50

A
B
C
D
E

30 ML no cal 60
15 ML cal 45

A
B
C
D

2 FR calc 30 min
4 FR no calc 60 min