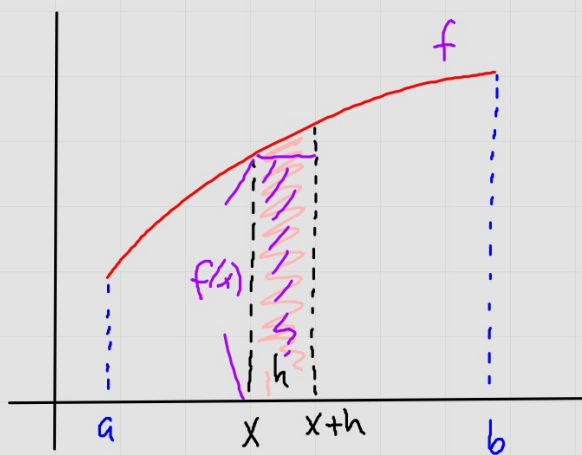


The First Fundamental Theorem of Calculus



let $A(x)$ be a func. that yields
the area under f from
 a to x .

$$A(a) = 0$$

$A(b)$ area from a to b .

$$f(x) \cdot h \approx A(x+h) - A(x)$$

$$f(x) \approx \frac{A(x+h) - A(x)}{h}$$

$$\lim_{h \rightarrow 0} f(x) = \lim_{h \rightarrow 0} \frac{A(x+h) - A(x)}{h}$$

$$f(x) = A'(x)$$

$$\int A'(x) dx = \int f(x) dx$$

$$A(x) = \int f(x) dx$$

Let $F(x)$ be another anti. of f .

$$A(x) = F(x) + C$$

$$A(a) = F(a) + C$$

$$0 = F(a) + C$$

$$C = -F(a)$$

$$A(x) = F(x) - F(a)$$

$$A(b) = F(b) - F(a)$$

1st Fun.
Thm of Calc

$$\int_a^b f(x) dx = F(b) - F(a) \quad \text{where } F'(x) = f(x).$$

$$\int_1^3 x^2 dx = \left. \frac{1}{3} x^3 \right|_1^3 = \left(\frac{27}{3} \right) - \left(\frac{1}{3} \right) = \frac{26}{3}.$$

$$\begin{aligned} \int_0^{2\pi} \sin x dx &= -\cos x \Big|_0^{2\pi} \\ &= (-\cos 2\pi) - (-\cos 0) \\ &= (-1) - (-1) \\ &= 0 \end{aligned}$$

$$\begin{aligned}\int_1^3 (x^2 + x) dx &= \left[\frac{1}{3}x^3 + \frac{1}{2}x^2 \right]_1^3 \\&= \left(\frac{27}{3} + \frac{9}{2} \right) - \left(\frac{1}{3} + \frac{1}{2} \right) \\&= \frac{27}{3} + \frac{9}{2} - \frac{1}{3} - \frac{1}{2} \\&= \frac{26}{3} + \frac{12}{3} \\&= \frac{38}{3}\end{aligned}$$

$$\int_0^{\ln 3} e^x \sqrt{1+e^x} dx$$

$$u = 1 + e^x$$

$$du = e^x dx$$

$$x=0 \rightarrow u=2$$

$$x=\ln 3 \rightarrow u=4$$

$$\int_2^4 u^{1/2} du = \frac{2}{3} u^{3/2} \Big|_2^4 = \frac{2\sqrt{u^3}}{3} \Big|_2^4$$

$$= \left(\frac{16}{3} \right) - \left(\frac{4\sqrt{2}}{3} \right),$$

$$= \frac{16 - 4\sqrt{2}}{3}$$

$$\int_{-1}^4 |x-2| dx$$

$$|x-2| = \begin{cases} x-2 & x \geq 2 \\ 2-x & x < 2 \end{cases}$$

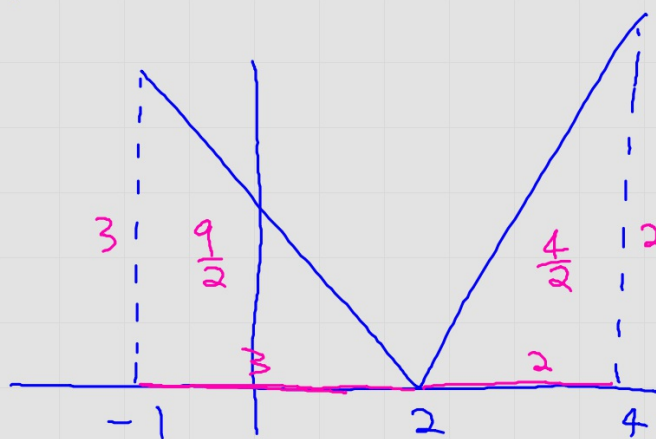
$$\int_{-1}^2 (2-x) dx + \int_2^4 (x-2) dx$$

DON'T DO THIS

$$= \left[2x - \frac{1}{2}x^2 \right]_{-1}^2 + \left[\frac{1}{2}x^2 - 2x \right]_2^4$$

$$= (\quad) - (\quad) + (\quad) - (\quad) .$$

$$\int_{-1}^4 |x-2| dx$$



$$\int_{-1}^4 |x-2| dx = \frac{9}{2} + \frac{4}{2} = \frac{13}{2}.$$