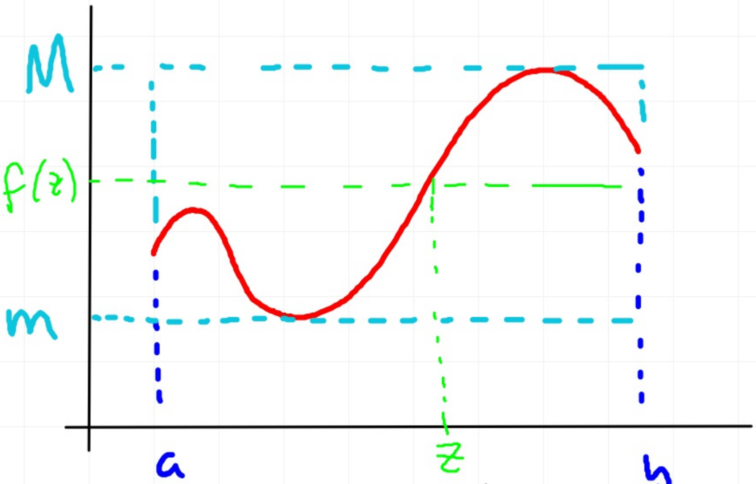
Average Value

Average value of a function on an interval...NOT to be confused with average rate of change in a function on an interval



$$m(b-a) \leq \int_a^b f(x)\,dx \leq M(b-a)$$

$$\int_a^b f(x)\,dx \in \left[m(b-a),\ M(b-a)\right]$$

$$f(z)(b-a) = \int_a^b f(x)\,dx$$

$$\underline{f(z)} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

$\Downarrow$

the avg. val. of f on [a,b]

Find a closed interval that contains the value of
$$\int_{\frac{1}{2}}^{4} (x^3 - 6x^2 + 9x + 1)\,dx$$ without integrating.

$$f(x) = x^3 - 6x^2 + 9x + 1$$
$$f'(x) = 3x^2 - 12x + 9$$
$f' \ \exists \ \forall x$

$f'(x) = 0 \to x = 1 \text{ or } x = 3$

$f(\frac{1}{2}) = 4.125$

$f(4) = 5 \to 5(4 - \frac{1}{2}) = \frac{35}{2}$

$f(1) = 5$

$f(3) = 1 \to 1(4 - \frac{1}{2}) = \frac{7}{2}$

$$\therefore \int_{\frac{1}{2}}^{4} (x^3 - 6x^2 + 9x + 1)\,dx \in \left[\frac{7}{2}, \frac{35}{2}\right].$$

rate of change

Find the avg. value of $f(x) = 4 - x^2$ on $[0, 2]$.

$$\frac{f(2) - f(0)}{2 - 0}$$

$$f_{avg} = \frac{1}{2-0}\int_0^2 (4 - x^2)\,dx$$

$$= \frac{1}{2}\left[4x - \frac{1}{3}x^3\right]_0^2$$

$$= \frac{1}{2}\left[\left(8 - \frac{8}{3}\right) - \left(0\right)\right]$$

$$= \frac{8}{3}.$$

$$f_{avg} = \frac{1}{b-a}\int_a^b f(x)\,dx$$ avg val of f on [a,b].

$$\frac{f(b)-f(a)}{b-a}$$ avg rate of change in f on [a,b]

$$V_{avg} = \frac{S(t_2)-S(t_1)}{t_2-t_1}$$

$$= \frac{1}{t_2-t_1}\int_{t_1}^{t_2} v(t)\,dt$$