Volumes of Solids with Known Cross Sections

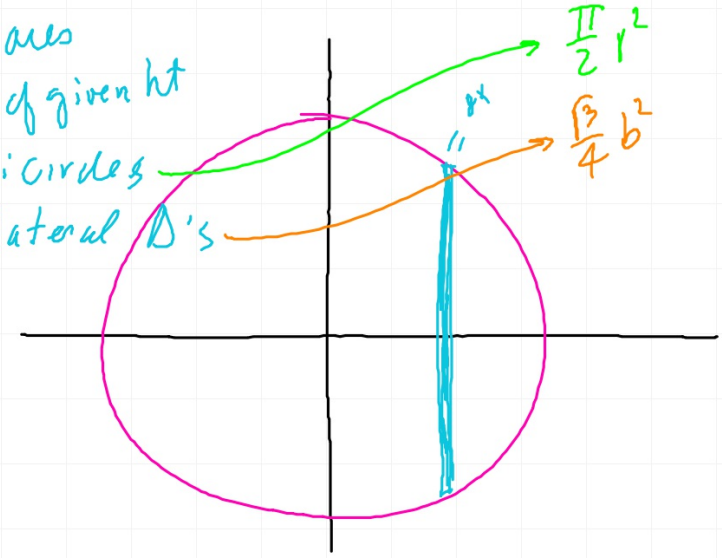
(Slicing)

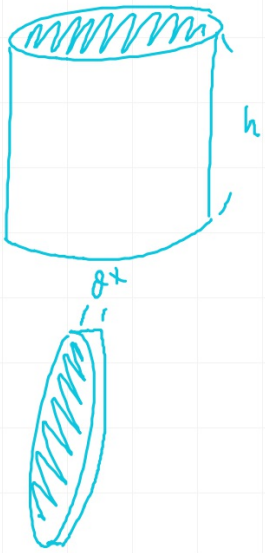
squares

rect of given ht

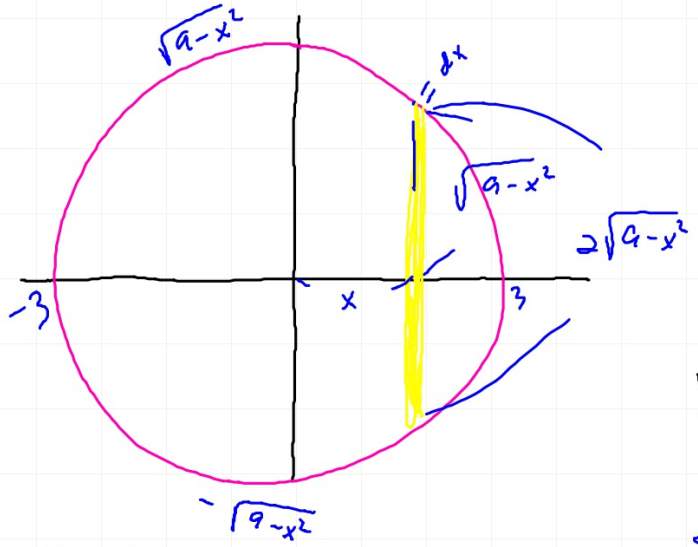
semicircles

equilateral Δ's





Base: $x^2+y^2=9 \rightarrow y=\pm\sqrt{9-x^2}$



Squares

$$V=\int_{-3}^{3}\left[2\sqrt{9-x^2}\right]^2 dx$$

Rectangle

$$V=\int_{-3}^{3}7\left(2\sqrt{9-x^2}\right)dx$$

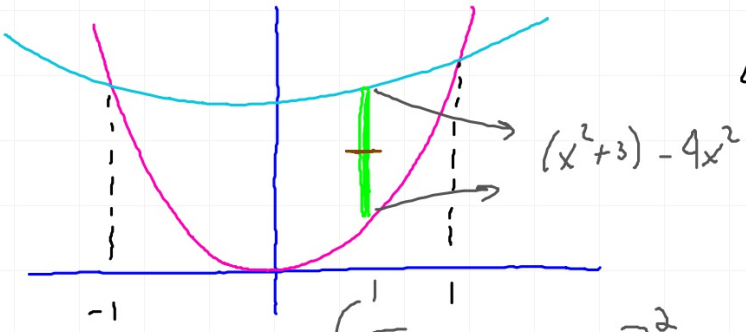
Eq. Δ

$$V=\frac{\sqrt{3}}{4}\int_{-3}^{3}\left[2\sqrt{9-x^2}\right]^2 dx$$

Semicirc

$$V=\frac{\pi}{2}\int_{-3}^{3}\left(\sqrt{9-x^2}\right)^2 dx$$

Base: $y = 4x^2$ and $y = x^2 + 3$ CS: Squares.



Isect

$4x^2 = x^2 + 3 \rightarrow x = -1 \text{ or } x = 1$

$$V_{\square} = \int_{-1}^{1} \left[(x^2+3) - 4x^2 \right]^2 dx = 9.600$$

SemiCirc

$$V = \frac{\pi}{2} \int_{-1}^{1} \left[\frac{(x^2+3) - 4x^2}{2} \right]^2 dx = 3.370$$