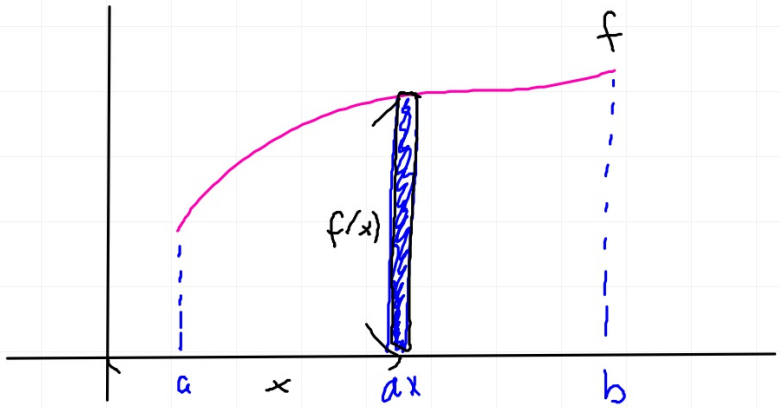
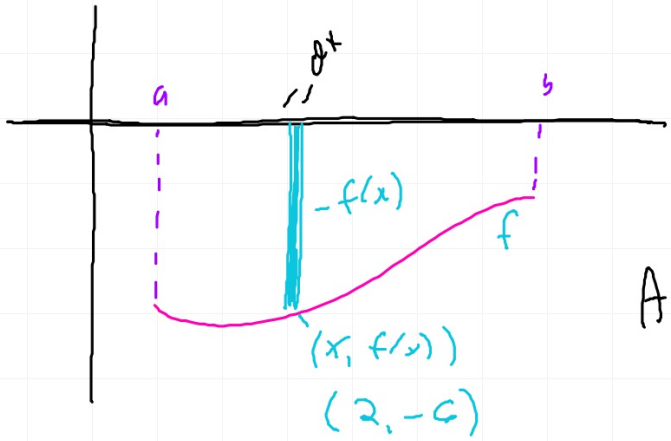
Area Under and Between Curves

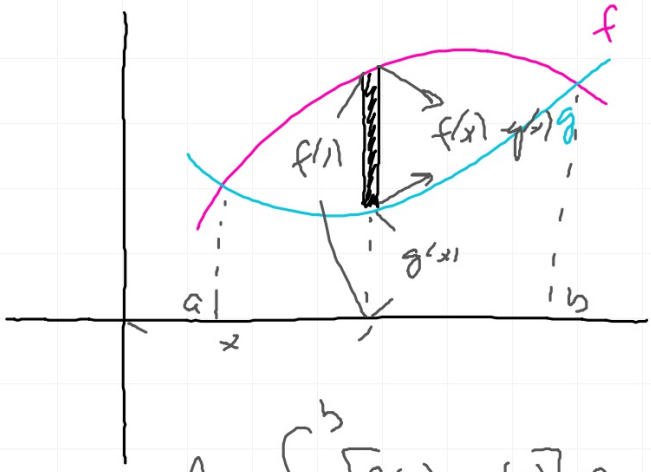


$$A = \int_a^b f(x)\,dx$$

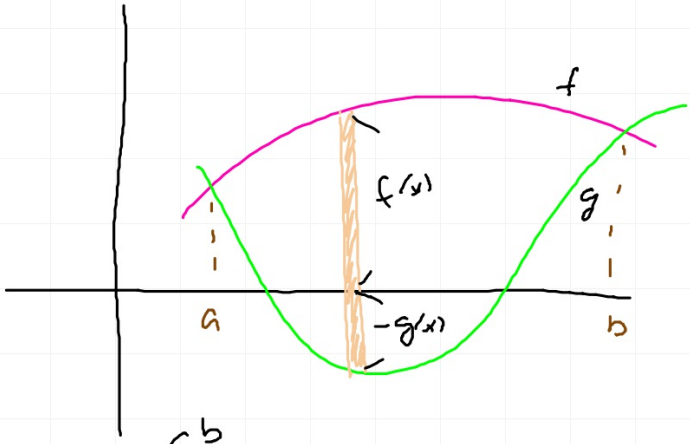


$$A = \int_a^b -f(x)\,dx = -\int_a^b f(x)\,dx$$

○ $-f(x)$



$$A = \int_a^b [f(x) - g(x)]\, dx$$



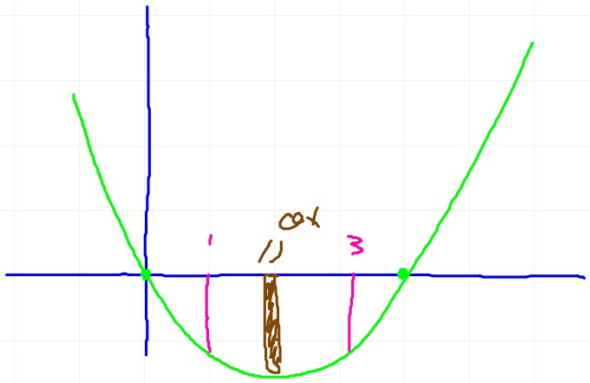
$$A = \int_a^b [f(x) - g(x)]\,dx$$

Bounds: $y = x^2 - 4x$, x-axis, $x = 1$, $x = 3$.

Zeros

$x^2 - 4x = 0$

$x = 0$ or $x = 4$



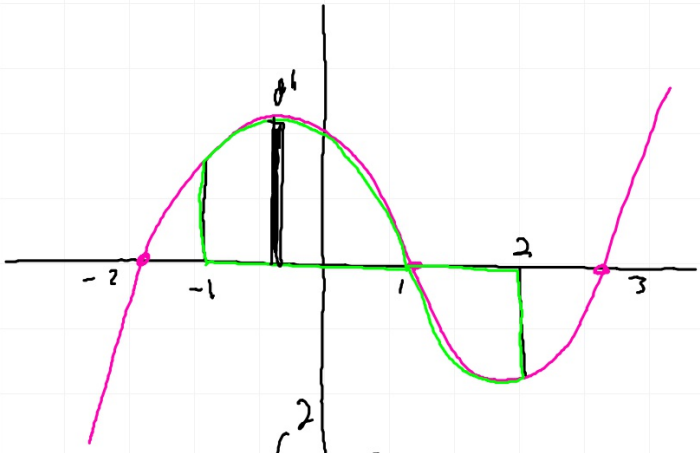
$$A = -\int_1^3 (x^2 - 4x)\,dx$$

Bounds: $y = x^3 - 2x^2 - 5x + 6$, x-axis, $x = -1$, $x = 2$.

Zeros

$x^3 - 2x^2 - 5x + 6 = 0$

$x = -2$ or $x = 1$ or $x = 3$



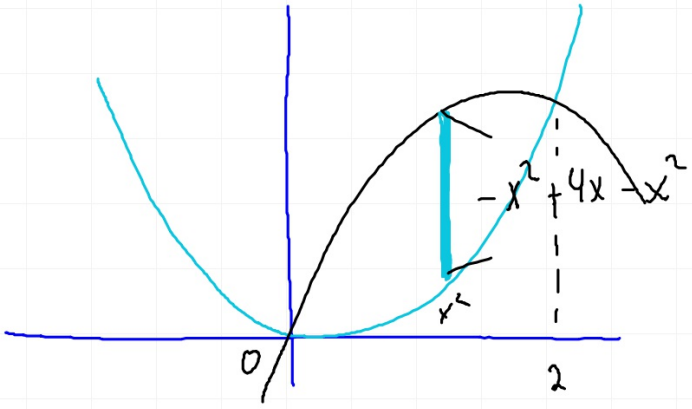
$$A = \int_{-1}^{1} (x^3 - 2x^2 - 5x + 6)\,dx - \int_{1}^{2} (x^3 - 2x^2 - 5x + 6)\,dx$$

Bands: $y = x^2$ and $y = -x^2 + 4x$

Isect

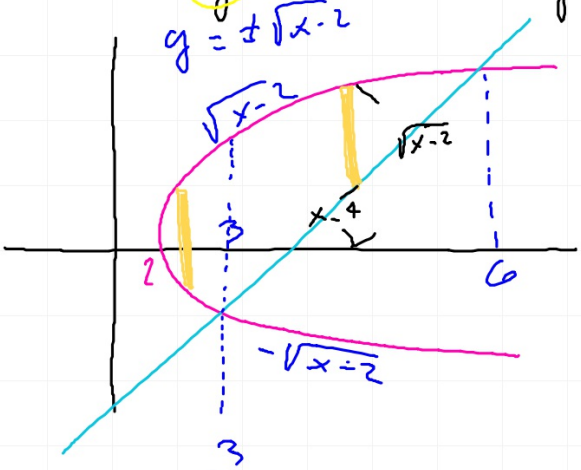
$x^2 = -x^2 + 4x$

$x = 0$ or $x = 2$



$$A = \int_0^2 [(-x^2 + 4x) - x^2]\,dx$$

Bounds: $y^2 = x-2$ and $y = x-4$



Isect

$y^2 + 2 = y + 4$

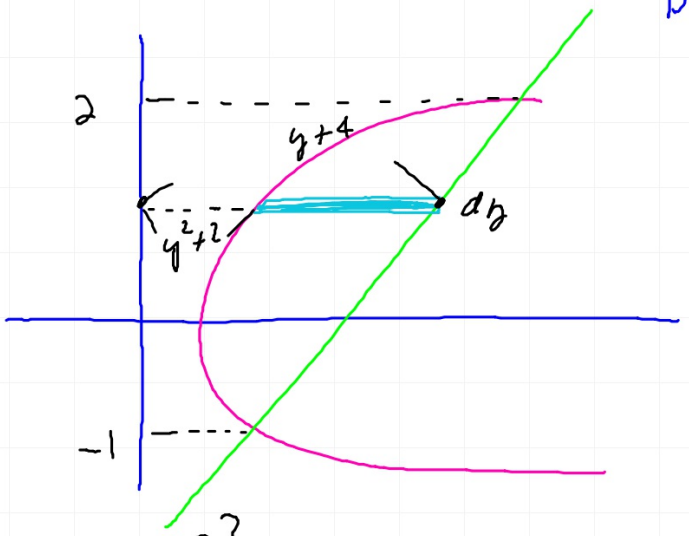
$y = -1$ or $y = 2$

$x = 3$ $x = 6$

$$A = \int_2^3 \left(\sqrt{x-2} + \sqrt{x+2}\right) dx + \int_3^6 \left[\sqrt{x-2} - (x-4)\right] dx$$

Bound: $y^2 = x-2$ and $y = x-4$

$x = y^2+2$ $\quad$ $x = y+4$



Set

$y^2+2 = y+4$

$y = -1$ and $y = 2$

$$A = \int_{-1}^{2} [(y+4)-(y^2+2)]\,dy$$