The Second Fundamental Theorem of Calculus

$$\boxed{\frac{d}{dx}\int_a^x f(t)\,dt = f(x)}$$

$$\frac{d}{dx}\int_3^x \sin t^2\,dt = \sin x^2$$

$$\frac{d}{dx}\int_a^x f(t)\,dt = \frac{d}{dx}\left[F(t)\Big|_a^x\right] = \frac{d}{dx}\left[F(x) - F(a)\right]$$

$$= F'(x)$$

$$= f(x)$$

$$\frac{d}{dx}\int_{3x}^{x^3}(t^2+4)\,dt = (x^6+4)(3x^2) - (9x^2+4)(3).$$

$$\frac{d}{dx}\int_{g(x)}^{h(x)} f(t)\,dt = \frac{d}{dx}\left[F(t)\Big|_{g(x)}^{h(x)}\right] = \frac{d}{dx}\left[F(h(x)) - F(g(x))\right]$$

$$= F'(h(x))h'(x) - F'(g(x))g'(x)$$

$$= f(h(x))h'(x) - f(g(x))g'(x)$$

$$\boxed{\frac{d}{dx}\int_{g(x)}^{h(x)} f(t)\,dt = f(h(x))\,h'(x) - f(g(x))\,g'(x)}$$

$$\frac{d}{dx}\int_{\sqrt{x}}^{x^3} (t^2-4)\,dt = (x^6-4)(3x^2) - (x-4)\left(\frac{1}{2\sqrt{x}}\right).$$

Given $F(x) = \int_2^x \sqrt{t-1}\, dt$ find $F(2)$, $F'(2)$, $\underline{F''(2)}$.

$F(2) = 0$

$F'(x) = \sqrt{x-1} \longrightarrow F'(2) = 1$

$F''(x) = \dfrac{1}{2\sqrt{x-1}} \longrightarrow F''(2) = \dfrac{1}{2}$

Given $F(x) + 3 = \int_s^x (t^2 - 1)\,dt$ $\quad F(s) \quad F'(s) \quad F''(s).$

$$F(x) = -3 + \int_s^x (t^2 - 1)\,dt$$

$$F(s) = -3$$

$$F'(x) = x^2 - 1 \longrightarrow F'(s) = 24$$

$$F''(x) = 2x \longrightarrow F''(s) = 10$$

Given $H(x) = \int_{5}^{x} \sec^3 t \, dt$, find $H(5) - H'(5)$.

$H(5) = 0$

$H'(x) = \sec^3 x$

$H'(5) = \sec^3 5$

$\therefore H(5) - H'(5) = 0 - \sec^3 5 = -\sec^3 5$.