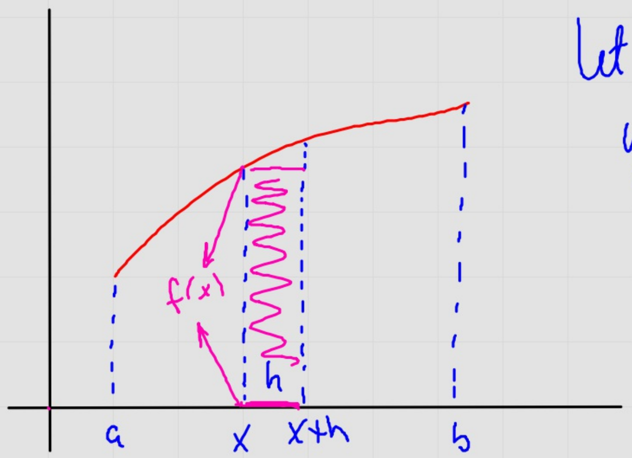
The First Fundamental Theorem of Calculus



Let $A(x)$ be a funct. that yields the area under f from a to x.

$A(a) = 0$

$A(b)$ area from a to b

$$f(x)\cdot h \approx A(x+h) - A(x)$$

$$f(x) \approx \frac{A(x+h) - A(x)}{h}$$

$$\lim_{h \to 0} f(x) = \lim_{h \to 0} \frac{A(x+h) - A(x)}{h}$$

$$f(x) = A'(x)$$

$$\int A'(x)\,dx = \int f(x)\,dx$$

$$A(x) = \int f(x)\,dx$$

Let $F(x)$ be another anti of f.

$$A(x) = F(x) + C$$
$$A(a) = F(a) + C$$
$$0 = F(a) + C$$
$$C = -F(a)$$

$$A(x) = F(x) - F(a)$$

$$A(b) = F(b) - F(a)$$

1st Fun. Thm of Calc.

$$\int_a^b f(x)\,dx = F(b) - F(a) \quad \text{where } F'(x) = f(x).$$

$$\int_1^3 x^2\,dx = \frac{1}{3}x^3\Big|_1^3 = \left(\frac{27}{3}\right) - \left(\frac{1}{3}\right) = \frac{26}{3}$$

$$\int_0^{2\pi} \sin x\,dx = -\cos x\Big|_0^{2\pi}$$

$$= (-\cos 2\pi) - (-\cos 0)$$

$$= (-1) - (-1)$$

$$= 0$$

$$\int_2^3 (x^2 + x)\,dx = \left[\frac{1}{3}x^3 + \frac{1}{2}x^2\right]_2^3$$

$$= \left(\frac{27}{3} + \frac{9}{2}\right) - \left(\frac{8}{3} + \frac{4}{2}\right)$$

$$= \frac{27}{3} + \frac{9}{2} - \frac{8}{3} - \frac{4}{2}$$

$$= \frac{19}{3} - \frac{5}{2}$$

$$= \frac{23}{6}$$

$$\frac{38 - 15}{6}$$

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$$\int_0^{\ln 3} e^x \sqrt{1+e^x}\, dx$$

$u = 1 + e^x$ $\qquad$ $x = 0 \to u = 2$

$du = e^x\, dx$ $\qquad$ $x = \ln 3 \to u = 4$

$$\int_2^4 u^{1/2}\, du = \frac{2}{3} u^{3/2} \Big|_2^4$$

$$= \frac{2\sqrt{u^3}}{3} \Big|_2^4$$

$$= \left(\frac{16}{3}\right) - \left(\frac{4\sqrt{2}}{3}\right)$$

$$= \frac{16 - 4\sqrt{2}}{3}$$

$$\int_{-1}^{4} |x-2|\, dx \qquad |x-2| = \begin{cases} x-2 & x \geq 2 \\ 2-x & x < 2 \end{cases}$$

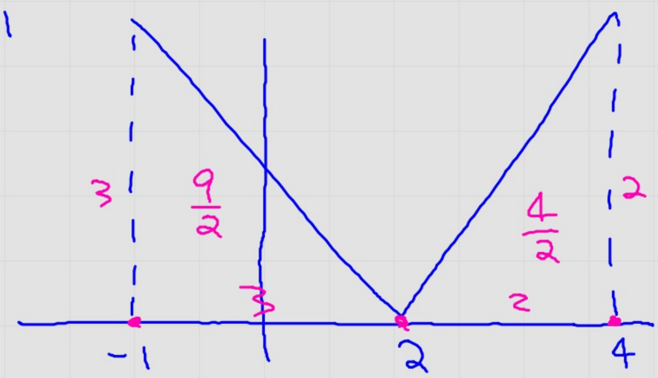
$$\int_{-1}^{2} (2-x)\, dx + \int_{2}^{4} (x-2)\, dx$$

DON'T DO THIS.

$$= \left[2x - \frac{1}{2}x^2\right]_{-1}^{2} + \left[\frac{1}{2}x^2 - 2x\right]_{2}^{4}$$

$$= (\qquad) - (\qquad) + (\qquad) - (\qquad)$$

$$\int_{-1}^{4} |x-2| \, dx$$



$$\int_{-1}^{4} |x-2| \, dx = \frac{9}{2} + \frac{4}{2} = \frac{13}{2}.$$