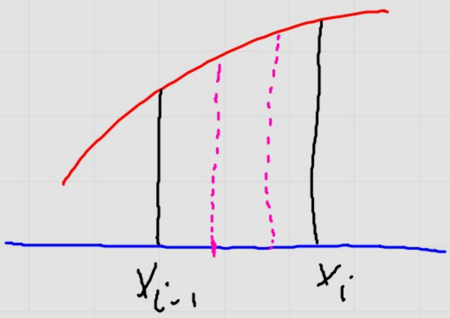
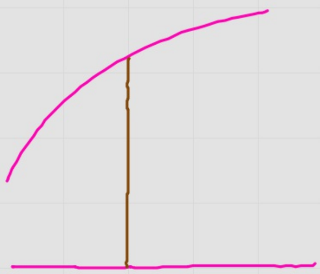
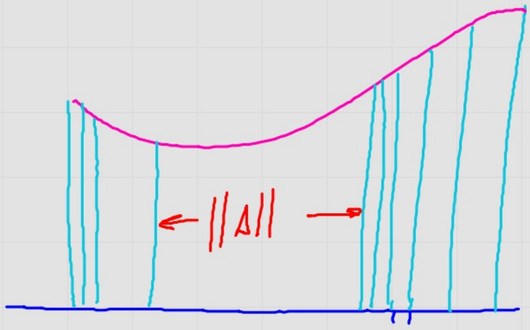
The Definite Integral

$$A = \lim_{n \to \infty} \sum_{i=1}^{n} f(c_i)\,\Delta x$$





$$A = \lim_{n \to \infty} \sum_{i=1}^{n} f(\xi_i)\,\Delta x$$



$\|\Delta\|$

"the norm"

$$A = \lim_{\|\Delta\| \to 0} \sum_{i=1}^{n} f(\xi_i)\,\Delta_i x$$

$$\int_a^b f(x)\,dx = \lim_{\|\Delta\| \to 0} \sum_{i=1}^{n} f(\xi_i)\,\Delta_i x$$

definite integral

Bounds: $f(x) = x^2 + 2$, $x = 2$, $x = 5$, x-axis

$$\int_2^5 (x^2 + 2)\,dx$$

Eval $\int_2^5 (x^2+2)\,dx$

$$\Delta x = \frac{3}{n} \qquad x_i = 2 + i\Delta x$$

$$\int_2^5 (x^2+2)\,dx = \lim_{x\to\infty} \sum_{i=1}^{n} f(2+i\Delta x)\Delta x$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left[\frac{18}{n} + \frac{36}{n^2} i + \frac{27}{n^3} i^2 \right]$$

$$= \lim_{n\to\infty} \left[18 + \frac{36}{n^2} \frac{n^2+n}{2} + \frac{27}{n^3} \frac{2n^3+3n^2+n}{6} \right]$$

$$= 18 + 18(1) + \frac{9}{2}(2)$$

$$= 45$$

Properties of the Def Int

$$\int_a^a f(x)\,dx = 0$$

$$\int_a^b k\,f(x)\,dx = k\int_a^b f(x)\,dx$$

$$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$$

$$\int_a^b k\,dx = k(b-a)$$

$$\int_a^b \left[f(x) \pm g(x) \right] dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$$

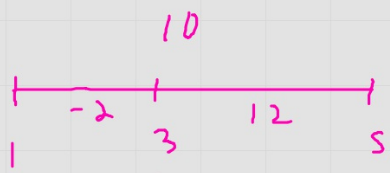
$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$ regardless of the order of a, b, c.

Given $\int_a^b f(x)\,dx = 8a - 2b$, find $\int_a^b [3f(x) - 6]\,dx$

$$3\int_a^b f(x)\,dx - \int_a^b 6\,dx$$

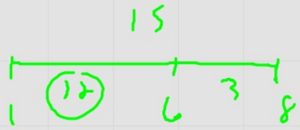
$$= 3(8a - 2b) - 6(b - a)$$
$$= 24a - 6b - 6b + 6a$$
$$= 30a - 12b.$$

Given $\int_1^5 f(x) = 10$ and $\int_1^3 f(x) = -2$, find $\int_3^5 f(x)\,dx$



$$\int_3^5 f(x)\,dx = 12.$$

Given $\int_1^8 f(x)\,dx = 15$ & $\int_6^8 f(x)\,dx = 3$, find $\int_1^6 [3f(x)-2)\,dx$



$$\int_1^6 f(x)\,dx = 12$$

$$3\int_1^6 f(x)\,dx - \int_1^6 2\,dx$$

$$= 3(12) - 2(5)$$

$$= 26.$$