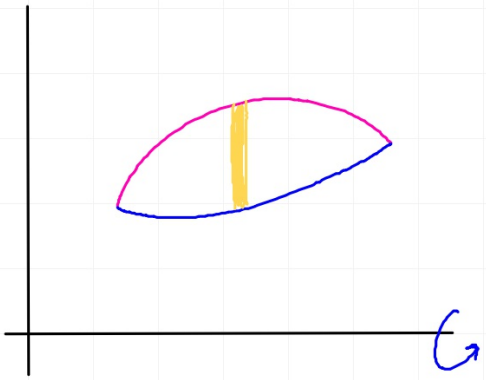
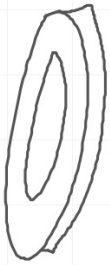
# *Volumes of Revolution--Disk/Washer Method*

Disk

$\pi r^2$ thickness

$dx$ $dy$

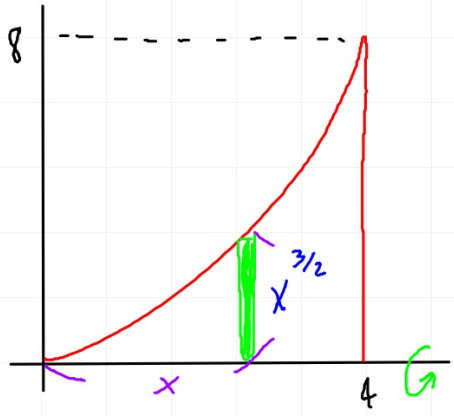




Washer

$\pi\left[r_o^2 - r_i^2\right]$ thickness

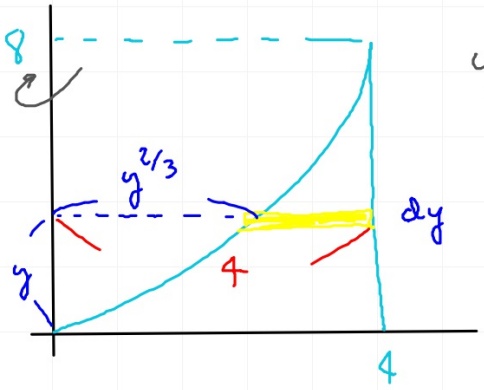
$dx$ $dy$




$y^2 = x^3$ ↻ x-axis

$y = x^{3/2}$

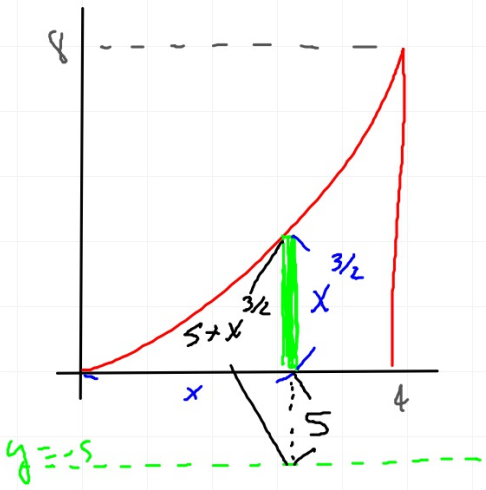
$$V_D = \pi \int_0^4 \left(x^{3/2}\right)^2 dx$$




$y^2 = x^3$ ↻ y-axis

$x = y^{2/3}$

$$V_W = \pi \int_0^8 \left[ (4)^2 - \left(y^{2/3}\right)^2 \right] dy$$

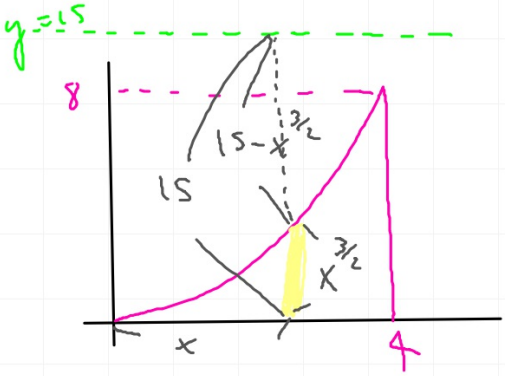



$y^2 = x^3 \Rightarrow y = -5$

$\downarrow$

$y = x^{3/2}$

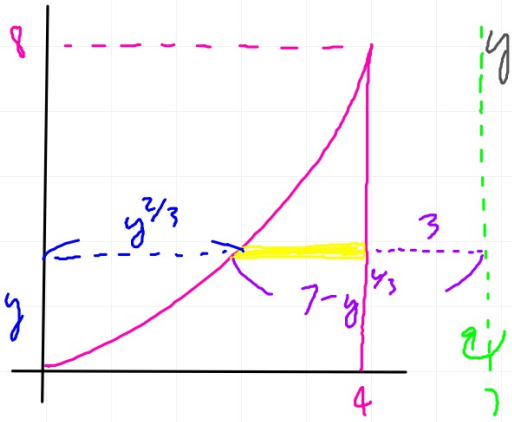
$$V_w = \pi \int_0^4 \left[ \left(5 + x^{3/2}\right)^2 - \left(\ 5\ \right)^2 \right] dx$$




$y^2 = x^3 \Rightarrow y = 15$

$y = x^{3/2}$

$$V_w = \pi \int_0^4 \left[ \left(\ 15\ \right)^2 - \left(15 - x^{3/2}\right)^2 \right] dx$$



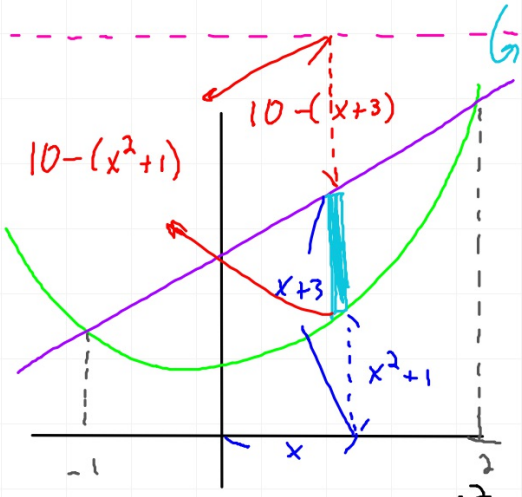

$y^2 = x^3$ e $x = 7$

$\downarrow$

$x = y^{2/3}$

$$V_w = \pi \int_0^8 \left[ (7 - y^{2/3})^2 - (3)^2 \right] dy$$

Bounds: $y = x^2+1$ and $y = x+3$ & $y = 10$




Isect

$x^2+1 = x+3 \rightarrow x = -1$ & $x = 2$

$$V = \pi \int_{-1}^{2} \left[ \left(10-(x^2+1)\right)^2 - \left(10-(x+3)\right)^2 \right] dx$$

$$y = 2x - x^2$$

$$y = -x^2 + 2x$$

$$y = -(x^2 - 2x + 1) + 1$$

$$y = -(x-1)^2 + 1$$

$$y - 1 = -(x-1)^2$$

$$(x-1)^2 = 1 - y$$

$$x - 1 = \sqrt{1-y}$$

$$x = 1 + \sqrt{1-y}$$