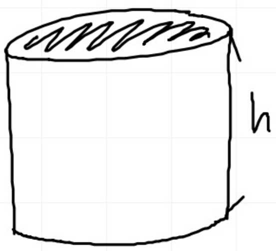
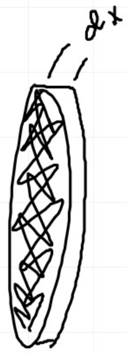
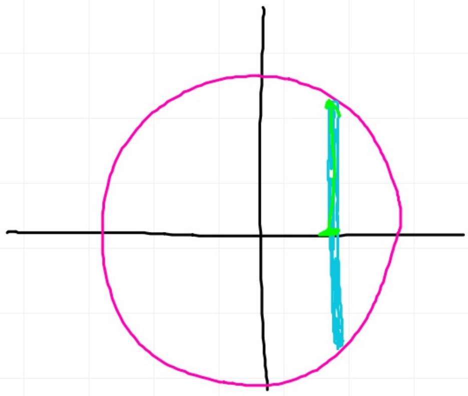
Volumes of Solids with Known Cross Sections

(Slicing)







$$V = \int_a^b (\text{area cross}) \begin{matrix} dx \\ dy \end{matrix}$$

squares

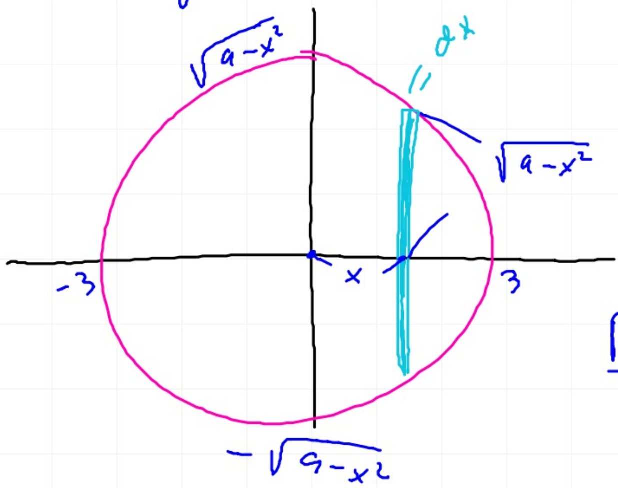
rect. of given

semicirc. $\rightarrow \frac{\pi}{2} r^2$

eqlat. $\Delta \rightarrow \frac{\sqrt{3}}{4} b^2$

Base: $x^2+y^2=9$ CS: squares

$y=\pm\sqrt{9-x^2}$



$$V_s = \int_{-3}^{3} \left[2\sqrt{9-x^2}\right]^2 dx$$

Rect of ht 5

$$V_{rect} = \int_{-3}^{3} 5\left(2\sqrt{9-x^2}\right) dx$$

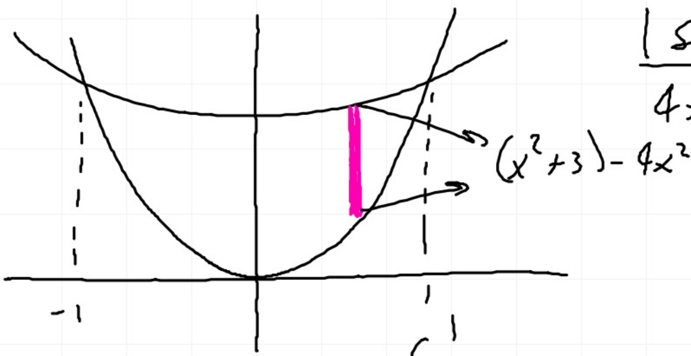
Eq Δ's

$$V = \frac{\sqrt{3}}{4}\int_{-3}^{3} \left(2\sqrt{9-x^2}\right)^2 dx$$

Semicircl

$$V = \frac{\pi}{2}\int_{-3}^{3} \left(\sqrt{9-x^2}\right) dx$$

Base: $y = 4x^2$ and $y = x^2+3$ semicri.



Isect

$4x^2 = x^2+3 \Rightarrow x=-1 \text{ a } x=1.$

$$V = \frac{\pi}{2}\int_{-1}^{1}\left[\frac{(x^2+3)-4x^2}{2}\right]^2 dx = 3.370$$

Square: $V = \int_{-1}^{1}\left[(x^2+3) - 4x^2\right]^2 dx$