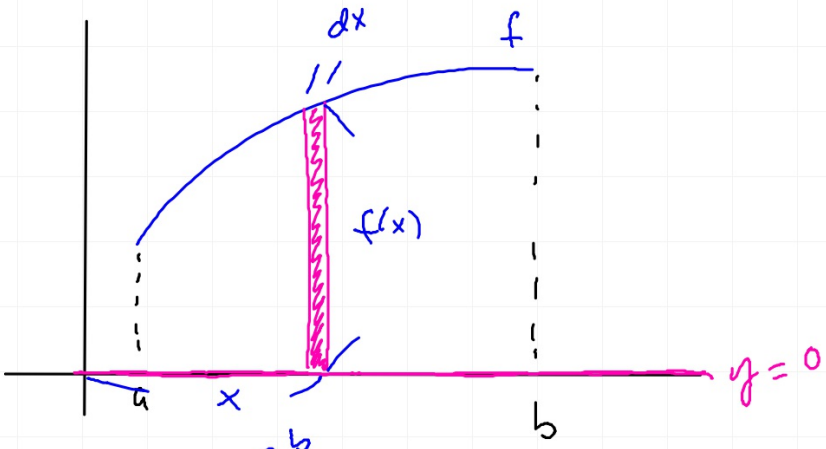
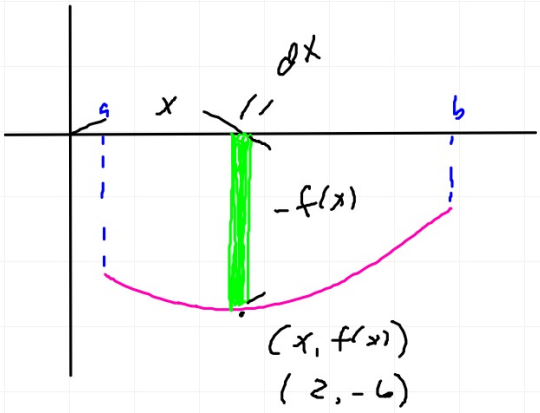
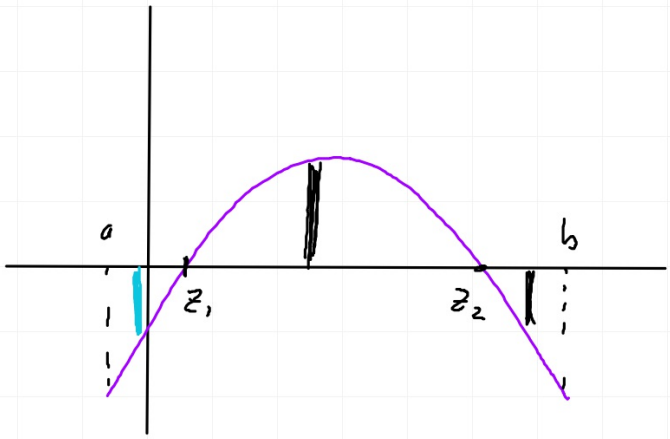
Area Under and Between Curves



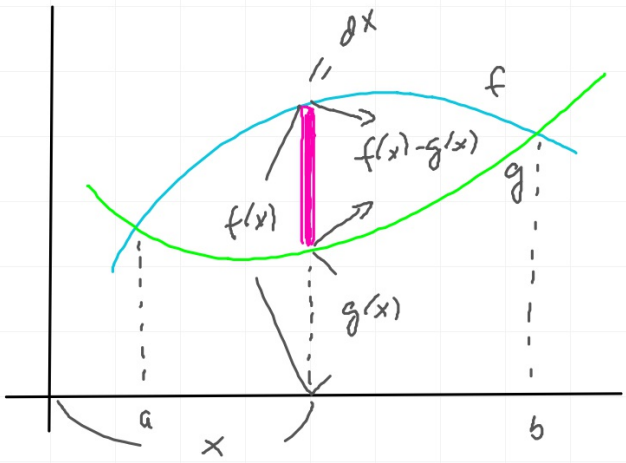
$$A = \int_a^b f(x)\,dx$$



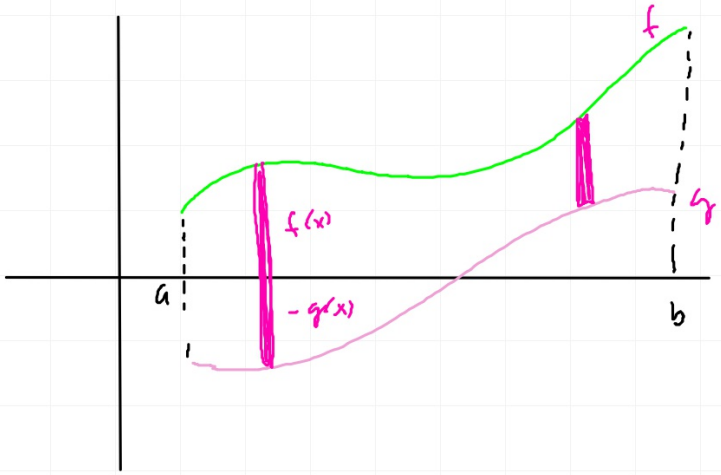
$$A = \int_a^b -f(x)\,dx = -\int_a^b f(x)\,dx$$



$$A = -\int_{a}^{z_1} f(x)\,dx + \int_{z_1}^{z_2} f(x)\,dx - \int_{z_2}^{b} f(x)\,dx$$

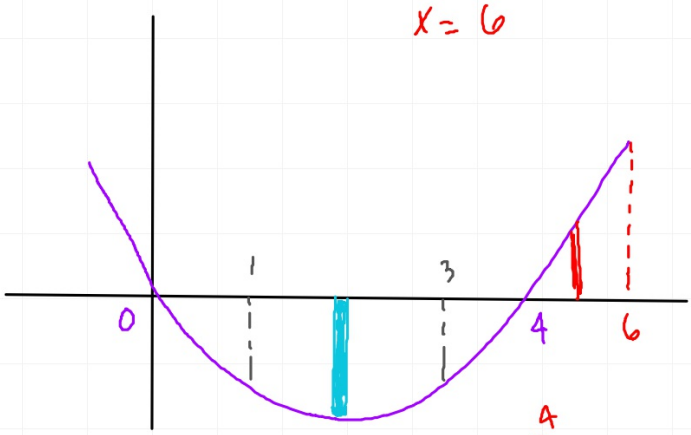


$$A = \int_a^b [f(x) - g(x)]\, dx$$


f
g
f(x)
-g(x)
a
b

Bounds: $y = x^2 - 4x$, x-axis, $x = 1$, $x = 3$.

$x = 6$



Zeros

$x^2 - 4x = 0$

$x = 0$ or $x = 4$

$$A = -\int_1^3 (x^2 - 4x)\,dx$$

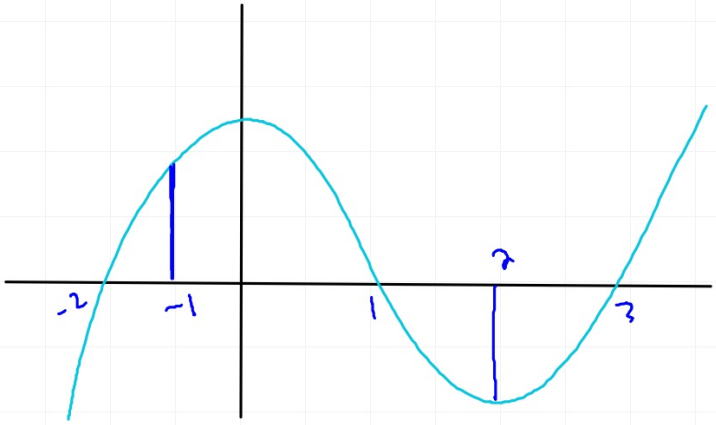
$$A = -\int_0^4 (x^2 - 4x)\,dx + \int_4^6 (x^2 - 4x)\,dx$$

Bounds: $y = x^3 - 2x^2 - 5x + 6$, x axis $x = -1$ and $x = 2$.

Zeros

$x^3 - 2x^2 - 5x + 6 = 0$

$x = -2$ or $x = 1$ or $x = 3$



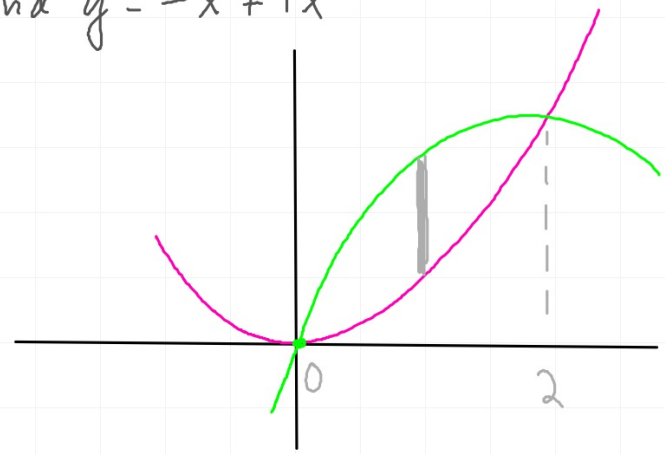
$$A = \int_{-1}^{1} (x^3 - 2x^2 - 5x + 6)\,dx - \int_{1}^{2} (x^3 - 2x^2 - 5x + 6)\,dx$$

Bounds: $y = x^2$ and $y = -x^2 + 4x$

Isect

$x^2 = -x^2 + 4x$

$x = 0$ or $x = 2$



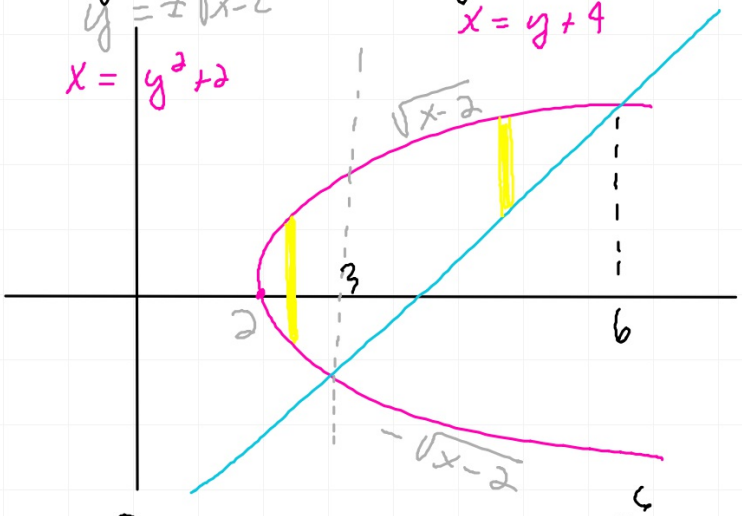
$$A = \int_0^2 \left[(-x^2+4x) - (x^2)\right] dx$$

Bounds: $y^2 = x-2$ and $y = x-4$

$y = \pm\sqrt{x-2}$

$x = y^2+2$

$x = y+4$



Isect

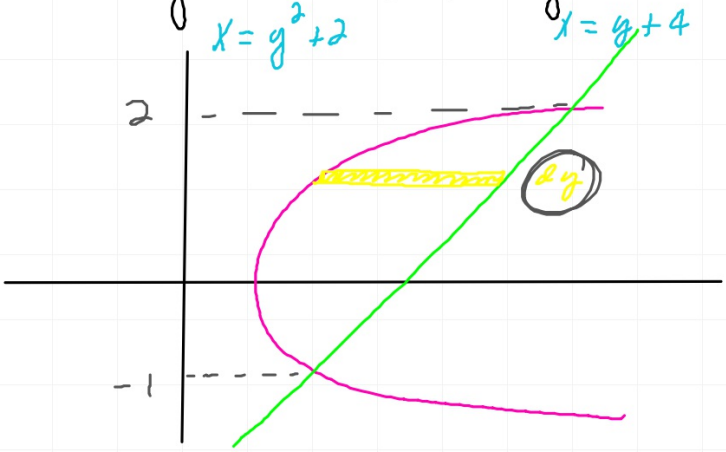
$y^2+2 = y+4$

$y = -1$ or $y = 2$

$x = 3$ $x = 6$

$$A = \int_2^3 \left[\left(\sqrt{x-2}\right) - \left(-\sqrt{x-2}\right)\right] dx + \int_3^6 \left[\left(\sqrt{x-2}\right) - (x-4)\right] dx$$

Bounds: $y^2 = x-2$ and $y = x-4$



Isect

$y^2+2 = y+4$

$y = -1$ or $y = 2$

$$A = \int_{-1}^{2} [(y+4) - (y^2+2)]\, dy$$