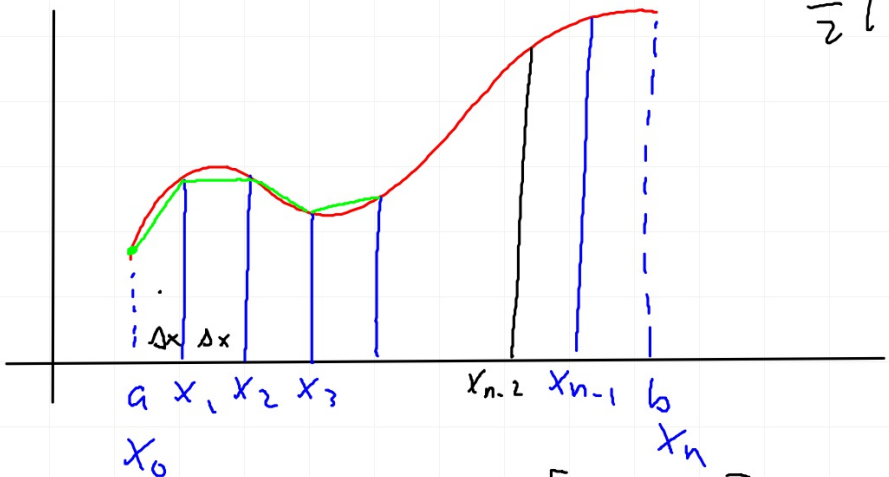
The Trapezoid Rule

(Another way to approximate area/estimate the value of a definite integral.)

$\frac{1}{2}(b_1+b_2)\,h$



$$\int_a^b f(x)\,dx \approx \frac{1}{2}\left[f(x_0)+f(x_1)\right]\Delta x + \frac{1}{2}\left[f(x_1)+f(x_2)\right]\Delta x$$
$$+ \frac{1}{2}\left[f(x_2)+f(x_3)\right]\Delta x + \cdots + \frac{1}{2}\left[f(x_{n-1})+f(x_n)\right]\Delta x$$

$$\int_a^b f(x)\,dx = \frac{b-a}{2n}\left[f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-1}) + f(x_n)\right]$$

$$\int_a^b f(x)\,dx \approx \frac{b-a}{2n}\sum_{i=0}^{n} k_i \cdot f(x_i)$$

$\hookrightarrow k_i = 1 \text{ or } 2$

Est. $\int_3^7 \sqrt{x-1}\,dx$ using $n=5$ and TR.

$\Delta x = \frac{4}{5} = .800$ $\qquad \frac{b-a}{2n} = .400$

	$f(x_i)$	k_i	$k_i f(x_i)$
$x_0 = 3$	1.414	1	1.414
$x_1 = 3.800$	1.673	2	3.347
$x_2 = 4.600$	1.897	2	3.795
$x_3 = 5.400$	2.098	2	4.195
$x_4 = 6.200$	2.280	2	4.561
$x_5 = 7$	2.449	1	2.449

$$\int_3^7 \sqrt{x-1}\,dx \approx (.4)(19.761)$$

$$\approx 7.904$$

TI 89

y1 = func.

seq(x, x, a, b, Δx) → a

y1(a) → b

{1, 2, 2, ... 2, 1} → k

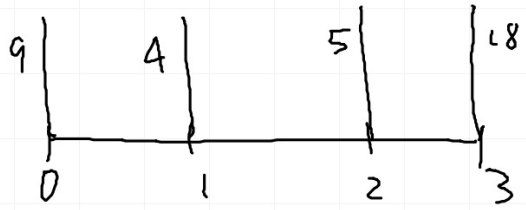
b * k → c *

sum(c)

* $\frac{b-a}{2n}$

① $\int_a^b f(x)\,dx > T$

②



$\frac{1}{2}(13)(1)$

$\frac{1}{2}(9)(1)$

$\frac{1}{2}(23)(1)$

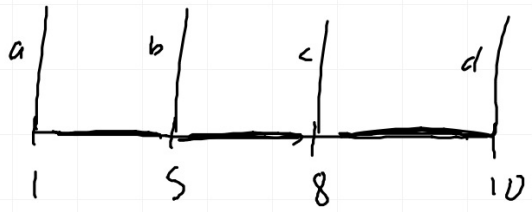
$\frac{13}{2} + \frac{9}{2} + \frac{23}{2} = \frac{45}{2}$

$x^3 - 6x + 9$

$1 - 6 + 9$

$8 - 12 + 9$

$27 - 18 + 9$



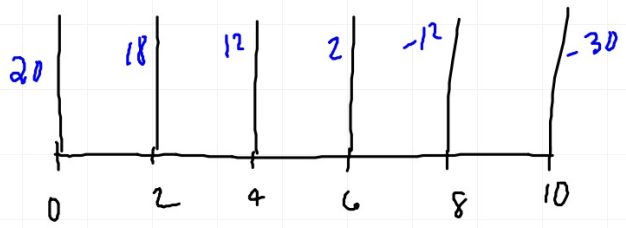
2 - Lost

$\frac{1}{2}(a+b)(4)$

$\frac{1}{2}(b+c)(3)$

$\frac{1}{2}(c+d)(2)$

④



$\frac{1}{2}(38)(2)$

$\frac{1}{2}(30)(2)$

$\frac{1}{2}(14)(2)$

$\frac{1}{2}(-10)(2)$

$\frac{1}{2}(-92)(2)$

$$I = \int_a^b f(x)\,dx$$

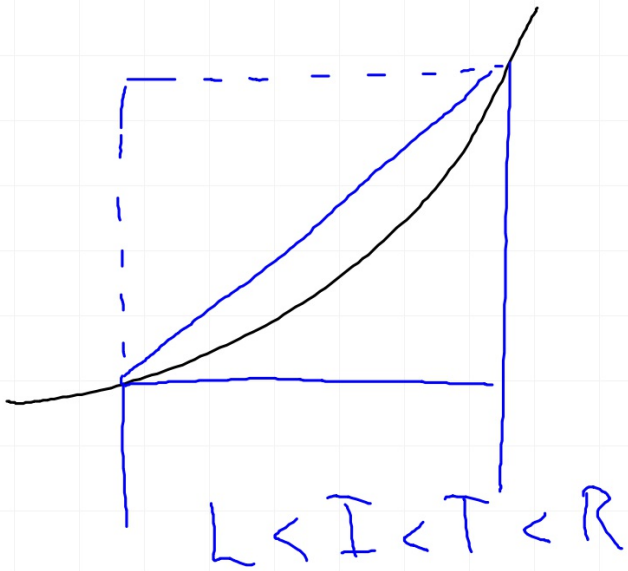
T trap

R right

L left

$f' > 0 \quad f'' > 0$

Put I, R, L, T in order from small to largest.



$L < I < T < R$

27 MC no cal 55 min
18 MC calcOK 50 min

30 MC no cal 60 min
15 MC calcOK 45 min

2 FR calcOK 30 min
4 FR no calc 60 min → same

A
B
C
D
E

A
B
C
D