The Second Fundamental Theorem of Calculus

$$\boxed{\frac{d}{dx}\int_a^x f(t)\,dt = f(x).}$$

$$\frac{d}{dx}\int_3^x t^2\,dt = x^2$$

$$\frac{d}{dx}\int_a^x f(t)\,dt = \frac{d}{dx}\left[F(t)\Big|_a^x\right] = \frac{d}{dx}\left[F(x)-F(a)\right]$$

$$= F'(x)$$

$$= f(x).$$

$$\frac{d}{dx}\int_{3x}^{x^2}(t^2-1)\,dt = (x^4-1)(2x)-(9x^2-1)(3).$$

$$\frac{d}{dx}\int_{g(x)}^{h(x)} f(t)\,dt = \frac{d}{dx}\left[F(t)\Big|_{g(x)}^{h(x)}\right]$$

$$= \frac{d}{dx}\left[F(h(x))-F(g(x))\right]$$

$$= F'(h(x))h'(x) - F'(g(x))g'(x)$$

$$= f(h(x))h'(x) - f(g(x))g'(x)$$

$$\frac{d}{dx}\int_{g(x)}^{h(x)} f(t)\,dt = f(h(x))h'(x) - f(g(x))g'(x)$$

$$\frac{d}{dx}\int_{5}^{x} \sqrt{t+2}\,dt = (\sqrt{x+2})(1) - \sqrt{7}(0)$$

$$= \sqrt{x+2}$$

$$F(x) = \int_2^x \sqrt{t-1}\,dt \quad \text{find } F(2),\ F'(2),\ F''(2).$$

$$F(2) = \int_2^2 \sqrt{t-1}\,dt = 0$$

$$F'(x) = \frac{d}{dx}\int_2^x \sqrt{t-1}\,dt = \sqrt{x-1} \longrightarrow F'(2) = \sqrt{1} = 1$$

$$F''(x) = \frac{1}{2\sqrt{x-1}} \longrightarrow F''(2) = \frac{1}{2}$$

$H(x) = \int_{s}^{x} \sec^3 t \, dt$ find $H(s) - H'(s)$.

$H(s) = 0$

$H'(x) = \sec^3 x$

$H'(s) = \sec^3 s$

$\therefore H(s) - H'(s) = -\sec^3 s$

$$F(x)+4=\int_3^x \sqrt{t-2}\,dt \qquad F(3) \quad F'(3) \quad F''(3)$$

$$F(x) = -4 + \int_3^x \sqrt{t-2}\,dt$$

$$F(3) = -4 + 0 = -4$$

$$F'(x) = \sqrt{x-2} \longrightarrow F'(3) = 1$$

$$F''(x) = \frac{1}{2\sqrt{x-2}} \longrightarrow F''(3) = \frac{1}{2}.$$

$$\frac{d}{dx}\int_{x}^{4} \sin t^2 \, dt = -\frac{d}{dx}\int_{4}^{x} \sin t^2 \, dt = -\sin x^2$$