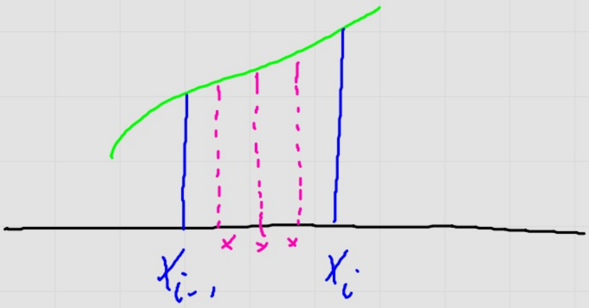
The Definite Integral

$$A = \lim_{n \to \infty} \sum_{i=1}^{n} f(c_i)\Delta x$$



$$A = \lim_{n \to \infty} \sum_{i=1}^{n} f(\xi_i) \Delta_i x$$

$\|\Delta\|$

"the norm"

$$A = \lim_{\|\Delta\| \to 0} \sum_{i=1}^{n} f(\xi_i) \Delta_i x$$

$$\boxed{\int_a^b f(x)\,dx} = \lim_{\|\Delta\| \to 0} \sum_{i=1}^{n} f(\xi_i)\,\Delta_i x$$

$\longrightarrow$ "the definite integral"

Bounds: $f(x) = x^2 + 2$, $x = 3$, $x = 5$, x-axis.

Evaluate: $\displaystyle\int_3^5 (x^2+2)\,dx$

Eval $\int_3^5 (x^2+2)\,dx$

$\Delta x = \frac{2}{n} \quad x_i = 3 + i\Delta x$

$$\int_3^5 (x^2+2)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(3+i\Delta x)\Delta x$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left[\frac{22}{n} + \frac{24}{n^2} i + \frac{8}{n^3} i^2\right]$$

$$= \lim_{n\to\infty} \left[22 + \frac{24}{n^2} \frac{n^2+n}{2} + \frac{8}{n^3} \frac{2n^3+3n^2+n}{6}\right]$$

$$= 22 + 12(1) + \frac{4}{3}(2)$$

$$= \frac{110}{3}$$

Properties of the Def. Integral

$$\int_a^a f(x)\,dx = 0$$

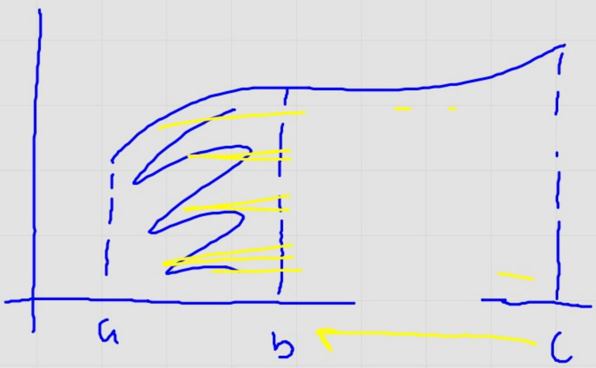
$$\int_a^b k\,f(x)\,dx = k\int_a^b f(x)\,dx$$

$$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$$

$$\int_a^b k\,dx = k(b-a)$$

$$\int_a^b \left[f(x) \pm g(x)\right] dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$$

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$ regardless of the order of a, b, c



Given $\int_a^b f(x)\,dx = 8a - 2b$ find $\int_a^b [5f(x) - 3]\,dx$.

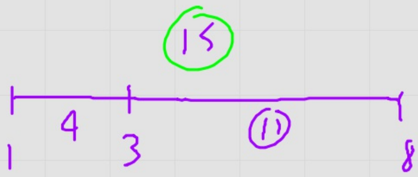
$$5\int_a^b f(x)\,dx - \int_a^b 3\,dx$$

$$= 5(8a-2b) - 3(b-a)$$

$$= 40a - 10b - 3b + 3a$$

$$= 43a - 13b$$

Given $\int_1^8 f(x)\,dx = 15$ and $\int_1^3 f(x)\,dx \overset{2a+3b}{=} 4$, find $\int_3^8 f(x)\,dx$.



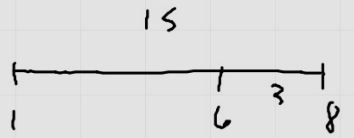
2a+3b

$$\int_3^8 f(x)\,dx = 11$$

$$15 - 2a - 3b$$

Given $\int_1^8 f(x)\,dx = 15$ and $\int_6^8 f(x)\,dx = 3$

find $\int_1^6 \left[3f(x)+2\right] dx$



$$\int_1^6 f(x)\,dx = 12$$

$$3\int_1^6 f(x)\,dx + \int_1^6 2\,dx$$

$$= 3(12) + 2(5) = 46.$$