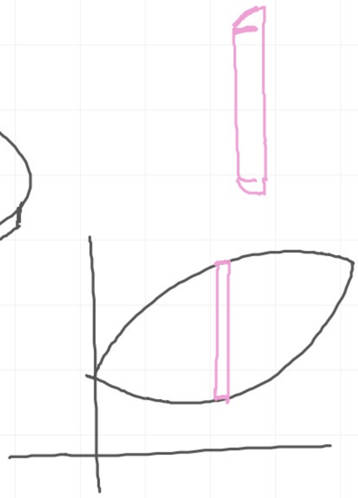
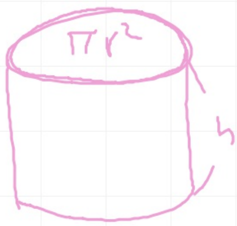
Volumes of Solids with Known Cross Sections

(Slicing)

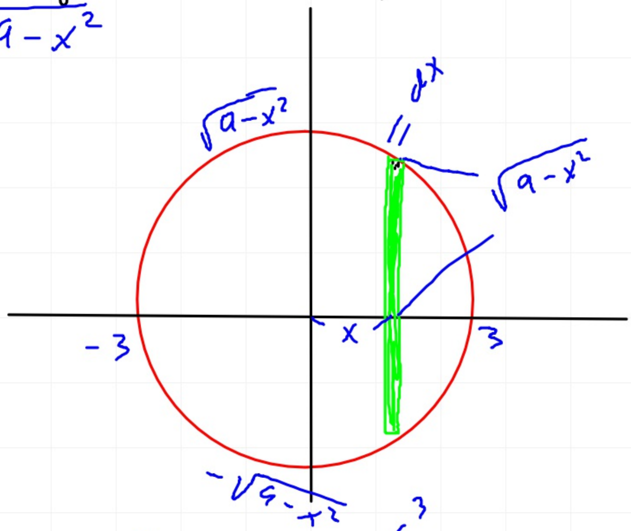


$V = \pi r^2 h$



Base: $x^2+y^2=9$ CS: squares

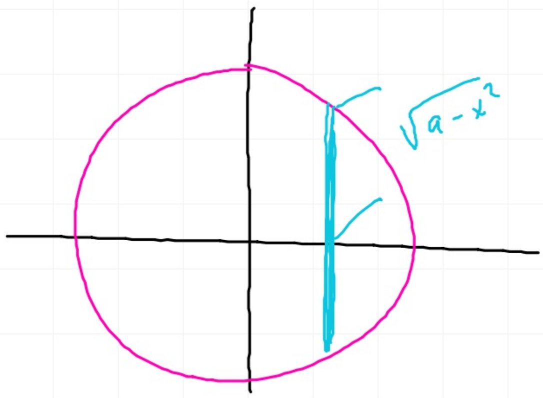
$y=\pm\sqrt{9-x^2}$



squares ✓
rectangles
equilateral △'s
$\left(\frac{\sqrt{3}}{4}b^2\right)$
semicircles
$\frac{1}{2}\pi r^2$

$$V=\int_{-3}^{3}\left(2\sqrt{9-x^2}\right)^2dx = 4\int_{-3}^{3}(9-x^2)\,dx = 8\int_{0}^{3}(9-x^2)\,dx$$

Base: $y = \pm\sqrt{9-x^2}$ rect. of ht 5.

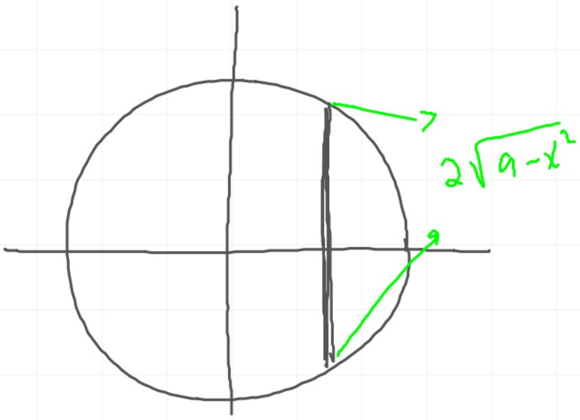


$$V = 5\int_{-3}^{3} 2\sqrt{9-x^2} = 10\int_{-3}^{3}\sqrt{9-x^2}\,dx$$

CS: semicircles

$$V = \frac{1}{2}\pi\int_{-3}^{3}\left(\sqrt{9-x^2}\right)^2 dx$$

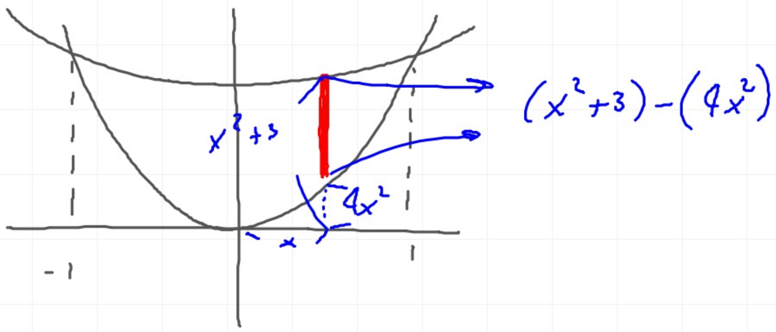
Base: $y = \pm\sqrt{9-x^2}$ cs: equi Δ's $\frac{\sqrt{3}}{4}b^2$



$$V = \frac{\sqrt{3}}{4}\int_{-3}^{3}\left(2\sqrt{9-x^2}\right)^2 dx$$

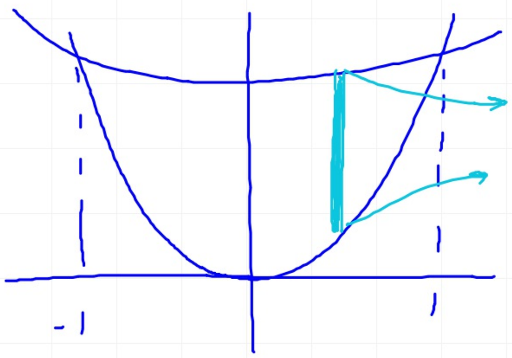
$$= 2\sqrt{3}\int_{0}^{3}(9-x^2)\,dx$$

Base: $y = 4x^2$ and $y = x^2 + 3$ CS: rect of ht 3



$$V = 3\int_{-1}^{1}\left[(x^2+3) - 4x^2\right]dx = 12$$

Base: $y = 4x^2$ & $y = x^2 + 3$ CS: semi $\left(\frac{1}{2}\pi r^2\right)$



$(x^2+3) - 4x^2$

$$V = \frac{\pi}{2}\int_{-1}^{1}\left[\frac{(x^2+3)-4x^2}{2}\right]^2 dx$$