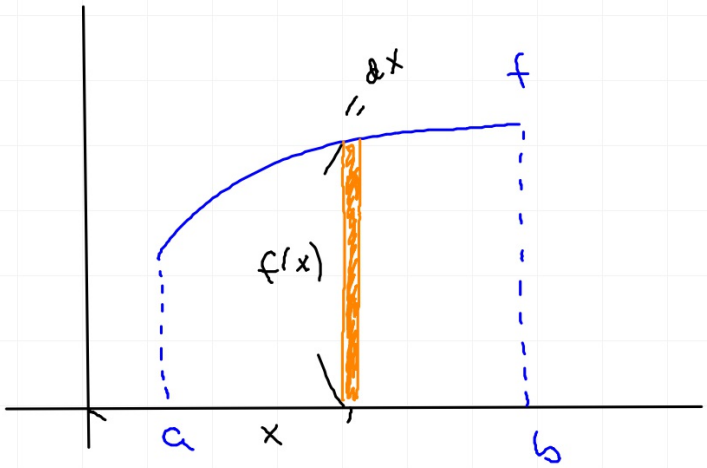
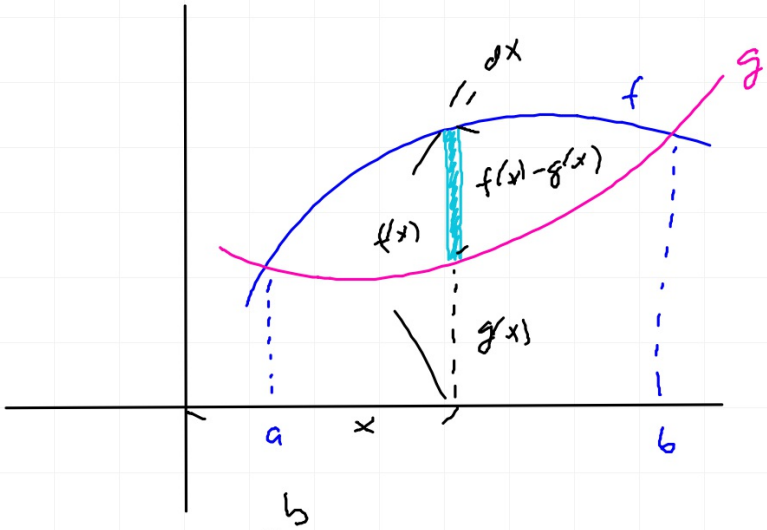
Area Under and Between Curves



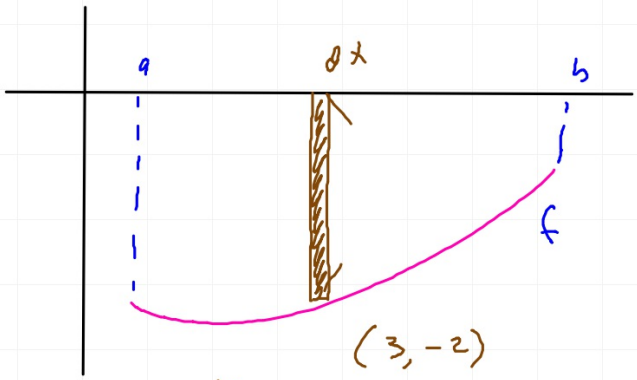
$$A = \int_a^b f(x)\,dx$$



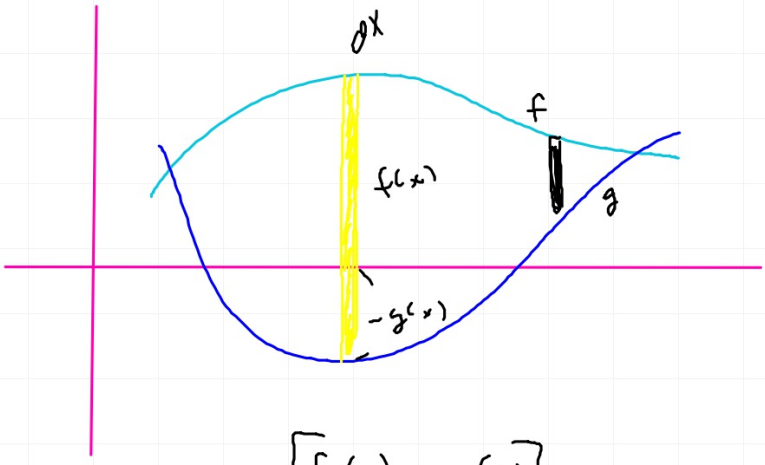
1 sect

$f(x) = g(x)$

$$A = \int_a^b \left[f(x) - g(x) \right] dx$$

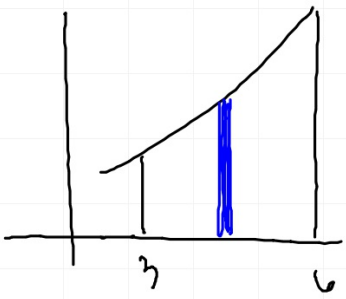


$$\int_a^b [0 - f(x)]\,dx = -\int_a^b f(x)\,dx$$



$$[f(x) - g(x)]$$

Bounds: $y = x\sqrt{x^2-5}$, x-axis, $x=3$, $x=6$



$$A = \int_3^6 x\sqrt{x^2-5}\,dx$$

$u = x^2 - 5$

$du = 2x\,dx$

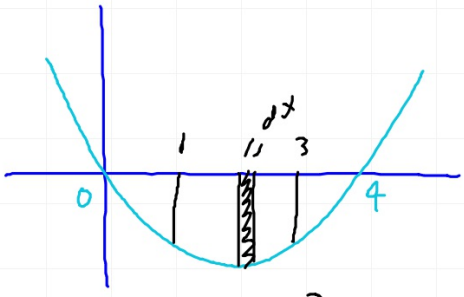
$\frac{1}{2}du = x\,dx$

$x = 3 \to u = 4$

$x = 6 \to u = 31$

$$= \frac{1}{2}\int_4^{31} u^{1/2}\,du$$

Bounds: $y = x^2 - 4x$, x axis, $x = 1$, $x = 3$



Zeros

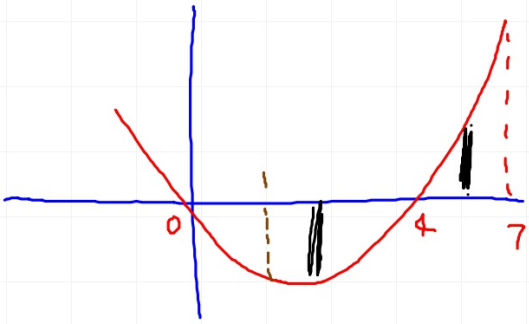
$x^2 - 4x = 0 \rightarrow x = 0$ or $x = 4$

$$A = -\int_1^3 (x^2 - 4x)\,dx$$

Bounds: $y = x^2 - 4x$, x-axis, $x = 1$, $x = 7$

Zeros

$x^2 - 4x = 0 \rightarrow x = 0$ or $x = 4$

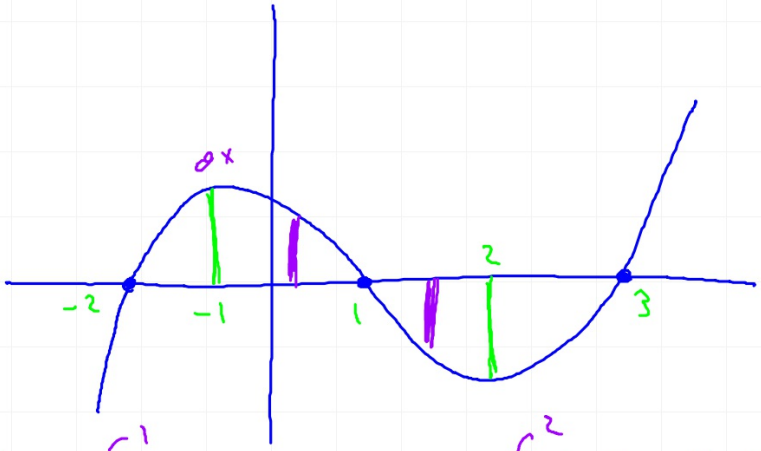


$$A = -\int_1^4 (x^2 - 4x)\,dx + \int_4^7 (x^2 - 4x)\,dx$$

Bounds: $y = x^3 - 2x^2 - 5x + 6$, x axis, $x = -1$, $x = 2$

Zeros

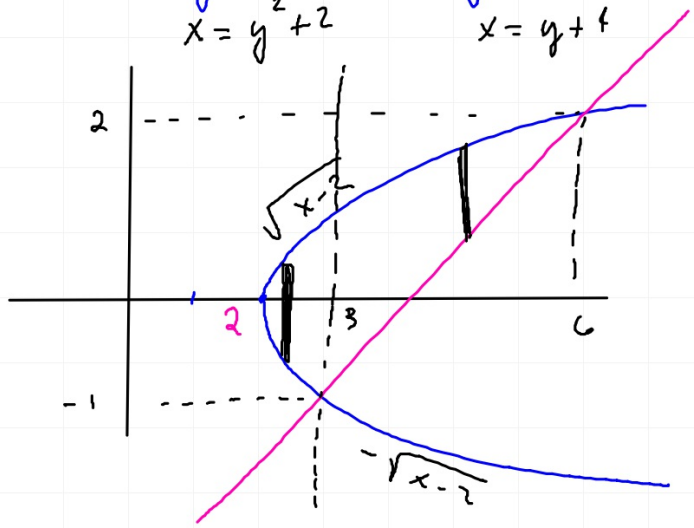
$x^3 - 2x^2 - 5x + 6 = 0 \rightarrow x = -2 \text{ or } x = 1 \text{ or } x = 3$



$$A = \int_{-1}^{1} (x^3 - 2x^2 - 5x + 6)\,dx - \int_{1}^{2} (x^3 - 2x^2 - 5x + 6)\,dx$$

Bounds: $y^2 = x-2$ $\quad y = x-4$

$x = y^2+2$ $\quad x = y+4$



Isect

$y^2+2 = y+4$

$y^2-y-2=0$

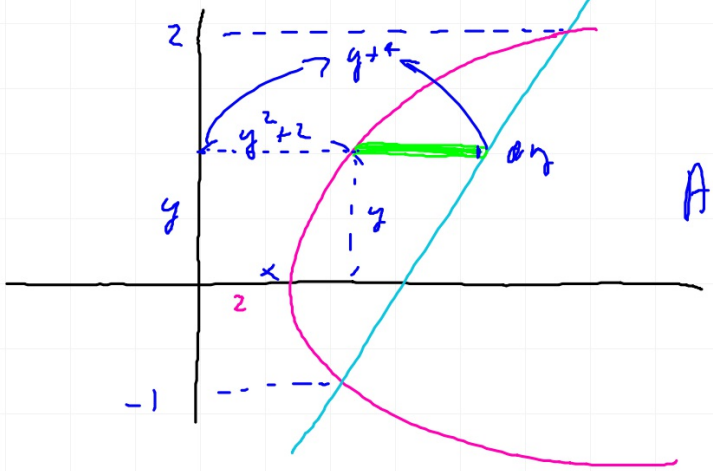
$y=2$ & $y=-1$

$x=6$ $\quad x=3$

$$A = \int_2^3 \left(\sqrt{x-2} + \sqrt{x-2}\right) dx + \int_3^6 \left[\sqrt{x-2} - (x-4)\right] dx$$

Bounds: $y^2 = x-2$ & $y = x-4$

$x = y^2+2$ $\quad$ $x = y+4$



Isect

$y^2+2 = y+4$

$y = -1$ & $y = 2$

$$A = \int_{-1}^{2} \left[(y+4)-(y^2+2)\right] dy$$