Left Rieman sum approx.

$f(x) = x^2 + 5x + 6$, $x = 1$ to $x = 6$, 4sub, L.S.

$\Delta x = \frac{5}{4}$

$x_0 = 1$
$x_1 = \frac{9}{4}$
$x_2 = \frac{7}{2}$
$x_3 = \frac{19}{4}$

$x_4 = 6$

$$A_L \approx \frac{5}{4} \sum_{i=0}^{3} f(x_i)$$

$$\approx \left(\frac{5}{4}\right)\left[12 + \frac{357}{16} + \frac{143}{4} + \frac{837}{16}\right]$$

$$\approx \frac{5}{4} \cdot \frac{979}{8}$$

$$\approx \frac{4895}{32}.$$

Right Rieman sum approx

$f(x) = x^2 + 5x + 6$, $x = 1$ to $x = 6$, 4 sub. R.S.

$\Delta x = \frac{5}{4} =$

$x_0 = 1$

$x_1 = 2.250$

$x_2 = 3.500$

$x_3 = 4.750$

$x_4 = 6$

$$A_R \approx \sum_{i=1}^{4} f(i)\,\Delta x$$

$$\approx \left(\frac{5}{4}\right)(182.375)$$

$$\approx 227.969$$

Midpoint Rieman sum approx

$f(x) = x^2 + 5x + 6$, $x = 1$ to $x = 5$, 5 sub, M.S.

$\Delta x = \frac{4}{5} = .800$

$x_0 = 1$

$x_1 = 1.800 \longrightarrow m_1 = 1.400$

$x_2 = 2.600 \longrightarrow m_2 = 2.200$

$x_3 = 3.400 \longrightarrow m_3 = 3$

$x_4 = 4.200 \longrightarrow m_4 = 3.800$

$x_5 = 5 \longrightarrow m_5 = 4.600$

$$A_M \approx \sum_{i=1}^{5} f(m_i)\,\Delta x$$

$$\approx (.8)(156.400)$$

$$\approx 125.120$$

Exact area via limit of Rieman sum

Bounds: $f(x) = x^2 + x$, $x = 2$, $x = 5$, x-axis

$$\Delta x = \frac{3}{n} \quad x_i = 2 + i\Delta x \checkmark$$

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} f(2 + i\Delta x)\Delta x \checkmark$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left[6\Delta x + 5i(\Delta x)^2 + i^2(\Delta x)^3 \right]$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left[\frac{18}{n} + \frac{45}{n^2} i + \frac{27}{n^3} i^2 \right] \checkmark$$

$$= \lim_{n\to\infty} \left[18 + \frac{45}{n^2} \frac{n^2+n}{2} + \frac{27}{n^3} \frac{2n^3+3n^2+n}{6} \right] \checkmark$$

$$= 18 + \frac{45}{2}(1) + \frac{9}{2}(2)$$

$$= \frac{99}{2} \checkmark$$

Trapezoidal approx

Est. $\int_3^7 \sqrt{x-1}\,dx$ using $n=5$ and TR.

$\Delta x = \frac{4}{5} = .800 \qquad \frac{b-a}{2n} = .400$

	$f(x_i)$	k_i	$k_i f(x_i)$
$x_0 = 3$	1.414	1	1.414
$x_1 = 3.800$	1.673	2	3.347
$x_2 = 4.600$	1.897	2	3.795
$x_3 = 5.400$	2.098	2	4.195
$x_4 = 6.200$	2.280	2	4.561
$x_5 = 7$	2.449	1	2.449

$\int_3^7 \sqrt{x-1}\,dx \approx (.4)(19.761)$

≈ 7.904

Never start an answer off with an = sign!

1. $\int_2^3 x e^{x^2} \, dx$

$$\int_2^4 e^u \, du = e^u \Big|_2^4$$

$$= e^4 - e^2$$

$u = x^2$

$du = 2x \, dx$

$\frac{1}{2} du = x \, dx$

$x = 2 \rightarrow u = 3$

$x = 3 \Rightarrow u = 9$

2. $\int_1^5 e^{2x+3}\,dx$

$$\int_1^5 e^{2x+3}\,dx = \frac{1}{2}e^{2x+3}\Big|_1^5$$

$$= \frac{1}{2}e^{13} - \frac{1}{2}e^{5}$$