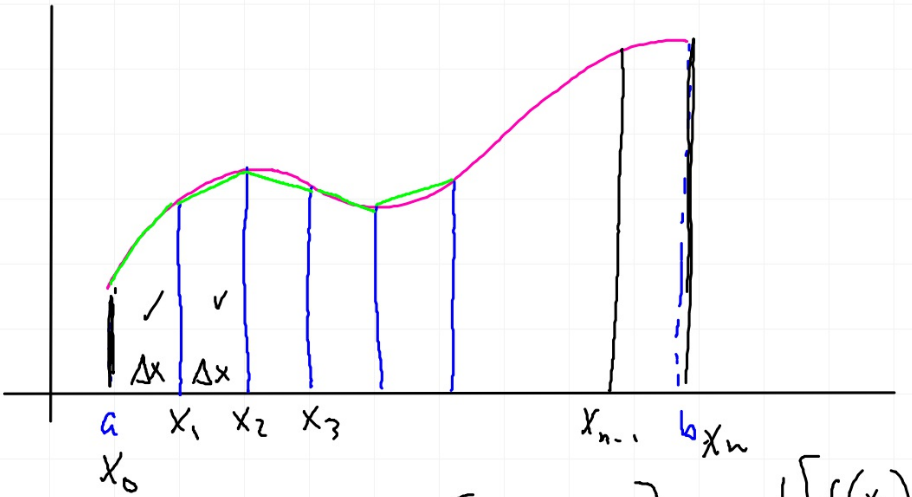
The Trapezoid Rule

(Another way to approximate area/estimate the value of a definite integral.)



$$A = \frac{1}{2}\left[f(x_0)+f(x_1)\right]\Delta x + \frac{1}{2}\left[f(x_1)+f(x_2)\right]\Delta x + \frac{1}{2}\left[f(x_2)+f(x_3)\right]\Delta x + \ldots$$

$$+ \frac{1}{2}\left[f(x_{n-1})+f(x_n)\right]\Delta x$$

$$A \approx \frac{1}{2}\Delta x \left[f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-1}) + f(x_n) \right]$$

$$A \approx \frac{b-a}{2n} \sum_{i=0}^{n} k_i \, f(x_i)$$

$\hookrightarrow k_i = 1$ o 2

Est. $\int_3^7 \sqrt{x-1}\,dx$ using TR with $n=5$.

$\Delta x = \frac{4}{5} = .800$

$\frac{b-a}{2n} = .400$

	b	h	c
	$f(x_i)$	k_i	$k f(x_i)$
$x_0 = 3$	1.414	1	1.414
$x_1 = 3.800$	1.673	2	3.347
$x_2 = 4.600$	1.897	2	3.795
$x_3 = 5.400$	2.098	2	4.195
$x_4 = 6.200$	2.280	2	4.561
$x_5 = 7$	2.449	1	2.449

$$\int_3^7 \sqrt{x-1}\,dx \approx (.4)(19.761) \approx 7.904.$$

y1 = func.

$\mathrm{seq}(x, x, a, b, \Delta x) \to a$

$y1(a) \to b$

$\{1, 2, 2, \ldots 2, 1\} \to k$

$b * k \to c$

$\mathrm{sum}(c)$

$* \frac{b-a}{2n}$

(1) $\int_a^b f(x)\,dx > T$ f CD

(2) $n=3$ $[0,3]$ $\int_0^3 (x^3-6x+9)\,dx$

$1-6+9$

$8-12+9$

$27-18+9$

22

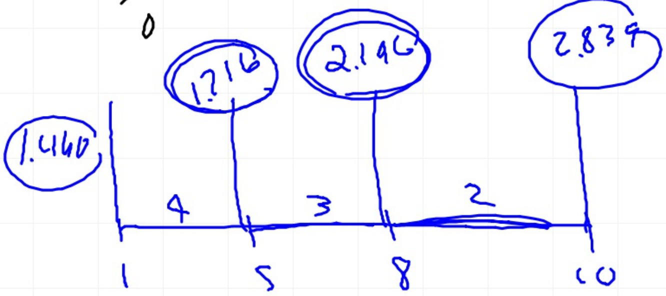
$\frac{1}{2}(9+4)(\cancel{1}) + \frac{1}{2}(4+5)(\cancel{1}) + \frac{1}{2}(5+18)(\cancel{1})$

$= \frac{1}{2}(13+9+23)$

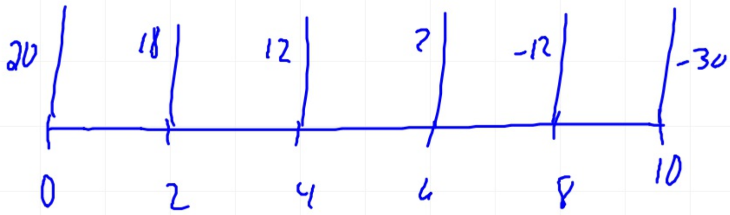
$= \frac{45}{2}$

$[1, 5]$ $[5, 8]$ $[8, 10]$

$$\int_0^{10} (2 - \cos x)\, dx$$



(4)

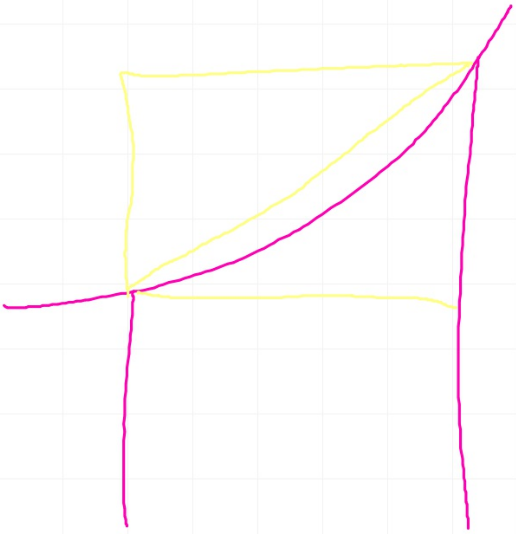


$$2(20+36+24+4-24-30)$$

$$\frac{1}{2}(38)(2) + \frac{1}{2}(30)2 + \frac{1}{2}(14)(2) + \frac{1}{2}(-10)2 + \frac{1}{2}(-42)2$$

(5) $I = \int_a^b f(x)\,dx \quad f' > 0$ and $f'' > 0$

L left, R right, T trap put L, R, T, I in order smallest to largest.



$L < I < T < R$