The Second Fundamental Theorem of Calculus

$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x) \qquad \frac{d}{dx}\int_3^x \sqrt{t-3}\,dt = \sqrt{x-3}.$$

$$\frac{d}{dx}\left[F(t)\Big|_a^x\right] = \frac{d}{dx}\left[F(x)-F(a)\right]$$

$$= f(x)$$

$$\frac{d}{dx}\int_{g(x)}^{h(x)} f(t)\,dt = f(h(x))\,h'(x) - f(g(x))\,g'(x)$$

$$\frac{d}{dx}\int_{3}^{x} t^2\,dt = (x^2)(1) - (9)(0) = x^2$$

$$\frac{d}{dx}\left[F(t)\Big|_{g(x)}^{h(x)}\right] = \frac{d}{dx}\left[F(h(x)) - F(g(x))\right]$$

$$= F'(h(x))\,h'(x) - F'(g(x))\,g'(x).$$

$$= f(h(x))\,h'(x) - f(g(x))\,g'(x)$$

$$\frac{d}{dx}\int_{\sqrt{x}}^{x^4} t^3\,dt = (x^{12})(4x^3) - (\sqrt{x})^3\frac{1}{2\sqrt{x}}.$$

$$= 4x^{15} - \frac{x}{2}$$

$$\frac{d}{dx}\int_{2}^{x}\sqrt{1-t}\,dt = \sqrt{1-x}$$

Given $F(x) = \int_2^x \sqrt{t-1}\,dt$ find $F(2)$, $F'(2)$, $F''(2)$.

$$F(2) = \int_2^2 \sqrt{t-1}\,dt = 0$$

$$F'(x) = \frac{d}{dx}\int_2^x \sqrt{t-1}\,dt$$

$$= \sqrt{x-1}$$

$$F'(2) = \sqrt{1} = 1$$

$$F''(x) = \frac{1}{2\sqrt{x-1}}$$

$$F''(2) = \frac{1}{2\sqrt{1}} = \frac{1}{2}.$$

$H(x) = \int_s^x \sec^3 t \, dt$, find $H(s) - H'(s)$.

$H(s) = 0$

$H'(x) = \sec^3 x \longrightarrow H'(s) = \sec^3 s$

$\therefore H(s) - H'(s) = 0 - \sec^3 s = -\sec^3 s.$

(13) Given $F(x) + 3 = \int_5^x (t-3)^3\,dt$, $F(5)$, $F'(5)$, $F''(5)$.

$$F(x) = -3 + \int_5^x (t-3)^3\,dt$$

$$F(5) = -3$$

$$F'(x) = (x-3)^3 \rightarrow F'(5) = 8$$

$$F''(x) = 3(x-3)^2(1) \rightarrow F''(5) = 12$$